Algebra Chapter R summary and review questions

Properties of the Real Numbers

Commutative:	$a + b = b + a;$ $ab = ba$
Associative:	$a + (b + c) =$ $(a + b) + c;$ $a(bc) = (ab)c$
Additive Identity:	$a + 0 = 0 + a = a$
Additive Inverse:	$-a + a =$ $a + (-a) = 0$
Multiplicative Identity:	$a \cdot 1 = 1 \cdot a = a$
Multiplicative Inverse:	$a \cdot \dfrac{1}{a} = \dfrac{1}{a} \cdot a = 1$ $(a \neq 0)$
Distributive:	$a(b + c) = ab + ac$

Absolute Value

For any real number a,

$$|a| = \begin{cases} a, & \text{if } a \geq 0, \\ -a, & \text{if } a < 0. \end{cases}$$

Special Products of Binomials

$$(A + B)^2 = A^2 + 2AB + B^2$$
$$(A - B)^2 = A^2 - 2AB + B^2$$
$$(A + B)(A - B) = A^2 - B^2$$

Sum or Difference of Cubes

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$
$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

Properties of Exponents

For any real numbers a and b and any integers m and n, assuming 0 is not raised to a nonpositive power:

The Product Rule: $a^m \cdot a^n = a^{m+n}$

The Quotient Rule: $\dfrac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$

The Power Rule: $(a^m)^n = a^{mn}$

Raising a Product to a Power: $(ab)^m = a^m b^m$

Raising a Quotient to a Power:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0)$$

Properties of Radicals

Let a and b be any real numbers or expressions for which the given roots exist. For any natural numbers m and n $(n \neq 1)$:

If n is even, $\sqrt[n]{a^n} = |a|$.

If n is odd, $\sqrt[n]{a^n} = a$.

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$.

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $(b \neq 0)$.

$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$.

Rational Exponents

For any real number a and any natural numbers m and n, $n \geq 2$, for which $\sqrt[n]{a}$ exists,

$a^{1/n} = \sqrt[n]{a}$,

$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$, and

$a^{-m/n} = \dfrac{1}{a^{m/n}}$.

In Exercises 1–6, consider the numbers $-43.89, 12, -3,$ $-\frac{1}{5}, \sqrt{7}, \sqrt[3]{10}, -1, -\frac{4}{3}, 7\frac{2}{3}, -19, 31, 0.$

1. Which are integers?

2. Which are natural numbers?

3. Which are rational numbers?

4. Which are real numbers?

5. Which are irrational numbers?

6. Which are whole numbers?

7. Write interval notation for $\{x \mid -3 \le x < 5\}$.

Simplify.

8. $|-3.5|$ 9. $|16|$

10. Find the distance between -7 and 3 on the number line.

Calculate.

11. $5^3 - [2(4^2 - 3^2 - 6)]^3$ 12. $\dfrac{3^4 - (6 - 7)^4}{2^3 - 2^4}$

Convert to decimal notation.

13. 3.261×10^6 14. 4.1×10^{-4}

Convert to scientific notation.

15. 0.01432 16. $43,210$

Calculate. Write the answer using scientific notation.

17. $\dfrac{2.5 \times 10^{-8}}{3.2 \times 10^{13}}$

18. $(8.4 \times 10^{-17})(6.5 \times 10^{-16})$

Simplify.

19. $(7a^2b^4)(-2a^{-4}b^3)$ 20. $\dfrac{54x^6y^{-4}z^2}{9x^{-3}y^2z^{-4}}$

21. $\sqrt[4]{81}$ 22. $\sqrt[5]{-32}$

23. $\dfrac{b - a^{-1}}{a - b^{-1}}$

24. $\dfrac{\dfrac{x^2}{y} + \dfrac{y^2}{x}}{y^2 - xy + x^2}$

25. $\left(\sqrt{3} - \sqrt{7}\right)\left(\sqrt{3} + \sqrt{7}\right)$

26. $\left(5x^2 - \sqrt{2}\right)^2$

27. $8\sqrt{5} + \dfrac{25}{\sqrt{5}}$

28. $(x + t)(x^2 - xt + t^2)$

29. $(5a + 4b)(2a - 3b)$

30. $(5xy^4 - 7xy^2 + 4x^2 - 3) -$
    $(-3xy^4 + 2xy^2 - 2y + 4)$

Factor.

31. $x^3 + 2x^2 - 3x - 6$

32. $12a^3 - 27ab^4$

33. $24x + 144 + x^2$

34. $9x^3 + 35x^2 - 4x$

35. $8x^3 - 1$

36. $27x^6 + 125y^6$

37. $6x^3 + 48$

38. $4x^3 - 4x^2 - 9x + 9$

39. $9x^2 - 30x + 25$

40. $18x^2 - 3x + 6$

41. $a^2b^2 - ab - 6$

42. Divide and simplify:

$$\dfrac{3x^2 - 12}{x^2 + 4x + 4} \div \dfrac{x - 2}{x + 2}.$$

43. Subtract and simplify:

$$\dfrac{x}{x^2 + 9x + 20} - \dfrac{4}{x^2 + 7x + 12}.$$

Write an expression containing a single radical.

44. $\sqrt{y^5}\ \sqrt[3]{y^2}$

45. $\dfrac{\sqrt{(a+b)^3}\ \sqrt[3]{a+b}}{\sqrt[6]{(a+b)^7}}$

46. Convert to radical notation: $b^{7/5}$.

47. Convert to exponential notation:

$$\sqrt[8]{\frac{m^{32}n^{16}}{3^8}}\ .$$

48. Rationalize the denominator:

$$\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}.$$

49. How long is a guy wire that reaches from the top of a 17-ft pole to a point on the ground 8 ft from the bottom of the pole?

Solve.

50. $2x - 7 = 7$

51. $5x - 7 = 3x - 9$

52. $8 - 3x = -7 + 2x$

53. $6(2x - 1) = 3 - (x + 10)$

54. $y^2 + 16y + 64 = 0$

55. $x^2 - x = 20$

56. $2x^2 + 11x - 6 = 0$

57. $x(x - 2) = 3$

58. $y^2 - 16 = 0$

59. $n^2 - 7 = 0$

Multiply. Assume that all exponents are integers.

66. $(x^n + 10)(x^n - 4)$

67. $(t^a + t^{-a})^2$

68. $(y^b - z^c)(y^b + z^c)$

69. $(a^n - b^n)^3$

Factor.

70. $y^{2n} + 16y^n + 64$

71. $x^{2t} - 3x^t - 28$

72. $m^{6n} - m^{3n}$

Review Exercises: Chapter R

1. [R.1] $12, -3, -1, -19, 31, 0$ **2.** [R.1] $12, 31$
3. [R.1] $-43.89, 12, -3, -\frac{1}{5}, -1, -\frac{4}{3}, 7\frac{2}{3}, -19, 31, 0$
4. [R.1] All of them   **5.** [R.1] $\sqrt{7}, \sqrt[3]{10}$ **6.** [R.1] $12, 31, 0$
7. [R.1] $[-3, 5)$ **8.** [R.1] 3.5 **9.** [R.1] 16 **10.** [R.1] 10
11. [R.2] 117 **12.** [R.2] -10 **13.** [R.2] $3,261,000$
14. [R.2] 0.00041 **15.** [R.2] 1.432×10^{-2}
16. [R.2] 4.321×10^4 **17.** [R.2] 7.8125×10^{-22}
18. [R.2] 5.46×10^{-32} **19.** [R.2] $-14a^{-2}b^7$, or $\dfrac{-14b^7}{a^2}$
20. [R.2] $6x^9y^{-6}z^6$, or $\dfrac{6x^9z^6}{y^6}$ **21.** [R.6] 3 **22.** [R.6] -2
23. [R.5] $\dfrac{b}{a}$ **24.** [R.5] $\dfrac{x+y}{xy}$ **25.** [R.6] -4
26. [R.6] $25x^4 - 10\sqrt{2}x^2 + 2$ **27.** [R.6] $13\sqrt{5}$
28. [R.3] $x^3 + t^3$ **29.** [R.3] $10a^2 - 7ab - 12b^2$
30. [R.3] $8xy^4 - 9xy^2 + 4x^2 + 2y - 7$
31. [R.4] $(x + 2)(x^2 - 3)$
32. [R.4] $3a(2a + 3b^2)(2a - 3b^2)$ **33.** [R.4] $(x + 12)^2$
34. [R.4] $x(9x - 1)(x + 4)$
35. [R.4] $(2x - 1)(4x^2 + 2x + 1)$
36. [R.4] $(3x^2 + 5y^2)(9x^4 - 15x^2y^2 + 25y^4)$
37. [R.4] $6(x + 2)(x^2 - 2x + 4)$
38. [R.4] $(x - 1)(2x + 3)(2x - 3)$
39. [R.4] $(3x - 5)^2$ **40.** [R.4] $3(6x^2 - x + 2)$
41. [R.4] $(ab - 3)(ab + 2)$ **42.** [R.5] 3
43. [R.5] $\dfrac{x - 5}{(x + 5)(x + 3)}$ **44.** [R.6] $y^3\sqrt[6]{y}$
45. [R.6] $\sqrt[3]{(a + b)^2}$ **46.** [R.6] $b\sqrt[5]{b^2}$ **47.** [R.6] $\dfrac{m^4n^2}{3}$
48. [R.6] $\dfrac{x - 2\sqrt{xy} + y}{x - y}$ **49.** [R.6] About 18.8 ft
50. [R.7] 7 **51.** [R.7] -1 **52.** [R.7] 3 **53.** [R.7] $-\frac{1}{13}$
54. [R.7] -8 **55.** [R.7] $-4, 5$ **56.** [R.7] $-6, \frac{1}{2}$
57. [R.7] $-1, 3$ **58.** [R.7] $-4, 4$ **59.** [R.7] $-\sqrt{7}, \sqrt{7}$
60. Discussion and Writing [R.2] Anya is probably not following the rules for order of operations. She is subtracting 6 from 15 first, then dividing the difference by 3, and finally multiplying the quotient by 4. The correct answer is 7.
61. Discussion and Writing [R.2] When the number 4 is raised to a positive integer power, the last digit of the result is 4 or 6. Since the calculator returns $4.398046511 \times 10^{12}$, or 4,398,046,511,000, we can conclude that this result is an approximation.   **62.** [R.2] $553.67   **63.** [R.2] $606.92
64. [R.2] $942.54   **65.** [R.2] $857.57
66. [R.3] $x^{2n} + 6x^n - 40$ **67.** [R.3] $t^{2a} + 2 + t^{-2a}$
68. [R.3] $y^{2b} - z^{2c}$ **69.** [R.3] $a^{3n} - 3a^{2n}b^n + 3a^nb^{2n} - b^{3n}$
70. [R.4] $(y^n + 8)^2$ **71.** [R.4] $(x^t - 7)(x^t + 4)$
72. [R.4] $m^{3n}(m^n - 1)(m^{2n} + m^n + 1)$