Algebra
Section R.5

Text excerpt:

The Domain of a Rational Expression

The **domain** of an algebraic expression is the set of all real numbers for which the expression is defined. Since division by zero is not defined, any number that makes the denominator zero is not in the domain of a rational expression.

We can describe the domains [redacted] using *set-builder notation*. For example, we write "The set of all real numbers x such that x is not equal to 3" as

$$\{x \mid x \text{ is a real number } \textit{and } x \neq 3\}.$$

Similarly, we write "The set of all real numbers x such that x is not equal to -1 and x is not equal to 5" as

$$\{x \mid x \text{ is a real number } \textit{and } x \neq -1 \textit{ and } x \neq 5\}.$$

Find the domain of the rational expression.

3. $\dfrac{3x - 3}{x(x - 1)}$

7. $\dfrac{7x^2 - 28x + 28}{(x^2 - 4)(x^2 + 3x - 10)}$

1. $-\dfrac{3}{4}$

Text excerpt:

Simplifying, Multiplying, and Dividing Rational Expressions

To simplify rational expressions, we use the fact that

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \cdot 1 = \frac{a}{b}.$$

To multiply rational expressions, we multiply numerators and multiply denominators and, if possible, simplify the result. To divide rational expressions, we multiply the dividend by the reciprocal of the divisor and, if possible, simplify the result—that is,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{and} \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

Multiply or divide and, if possible, simplify.

9. $\dfrac{x^2 - y^2}{(x - y)^2} \cdot \dfrac{1}{x + y}$

15. $\dfrac{m^2 - n^2}{r + s} \div \dfrac{m - n}{r + s}$

19. $\dfrac{x^2 - y^2}{x^3 - y^3} \cdot \dfrac{x^2 + xy + y^2}{x^2 + 2xy + y^2}$

21. $\dfrac{(x - y)^2 - z^2}{(x + y)^2 - z^2} \div \dfrac{x - y + z}{x + y - z}$

Text excerpt:

Adding and Subtracting Rational Expressions

When rational expressions have the same denominator, we can add or subtract by adding or subtracting the numerators and retaining the common denominator. If the denominators differ, we must find equivalent rational expressions that have a common denominator. In general, it is most efficient to find the **least common denominator** (**LCD**) of the expressions.

> To find the least common denominator of rational expressions, factor each denominator and form the product that uses each factor the greatest number of times it occurs in any factorization.

Add or subtract and, if possible, simplify.

25. $\frac{3}{2a + 3} + \frac{2a}{2a + 3}$

27. $\frac{5}{4z} - \frac{3}{8z}$

35. $\frac{y}{y - 1} + \frac{2}{1 - y}$
(*Note*: $1 - y = -1(y - 1)$.)

29. $\frac{3}{x + 2} + \frac{2}{x^2 - 4}$

33. $\frac{3}{x + y} + \frac{x - 5y}{x^2 - y^2}$

Text excerpt:

Complex Rational Expressions

A **complex rational expression** has rational expressions in its numerator or its denominator or both.

To simplify a complex rational expression:

Method 1. Find the LCD of all the denominators within the complex rational expression. Then multiply by 1 using the LCD as the numerator and the denominator of the expression for 1.

Method 2. First add or subtract, if necessary, to get a single rational expression in the numerator and in the denominator. Then divide by multiplying by the reciprocal of the denominator.

Simplify

49. $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{\dfrac{1}{y} + \dfrac{1}{x}}$

55. $\dfrac{a - a^{-1}}{a + a^{-1}}$

Further problems for knowledge synthesis and capstone

Simplify

65. $\dfrac{(x+h)^2 - x^2}{h}$

69. $\left[\dfrac{\dfrac{x+1}{x-1}+1}{\dfrac{x+1}{x-1}-1}\right]^5$

Perform the indicated operations and, if possible, simplify.

73. $\dfrac{x^2-9}{x^3+27} \cdot \dfrac{5x^2-15x+45}{x^2-2x-3} + \dfrac{x^2+x}{4+2x}$

Exercise Set R.5

1. $\{x \mid x \text{ is a real number}\}$
3. $\{x \mid x \text{ is a real number } and\ x \neq 0\ and\ x \neq 1\}$
5. $\{x \mid x \text{ is a real number } and\ x \neq -5\ and\ x \neq 1\}$
7. $\{x \mid x \text{ is a real number } and\ x \neq -2\ and\ x \neq 2\ and\ x \neq -5\}$
9. $\frac{1}{x - y}$ **11.** $\frac{(x + 5)(2x + 3)}{7x}$ **13.** $\frac{a + 2}{a - 5}$ **15.** $m + n$
17. $\frac{3(x - 4)}{2(x + 4)}$ **19.** $\frac{1}{x + y}$ **21.** $\frac{x - y - z}{x + y + z}$ **23.** $\frac{3}{x}$ **25.** 1
27. $\frac{7}{8z}$ **29.** $\frac{3x - 4}{(x + 2)(x - 2)}$ **31.** $\frac{-y + 10}{(y + 4)(y - 5)}$
33. $\frac{4x - 8y}{(x + y)(x - y)}$ **35.** $\frac{y - 2}{y - 1}$ **37.** $\frac{x + y}{2x - 3y}$
39. $\frac{3x - 4}{(x - 2)(x - 1)}$ **41.** $\frac{5a^2 + 10ab - 4b^2}{(a + b)(a - b)}$
43. $\frac{11x^2 - 18x + 8}{(2 + x)(2 - x)^2}$, or $\frac{11x^2 - 18x + 8}{(x + 2)(x - 2)^2}$ **45.** 0
47. $\frac{x + y}{x}$ **49.** $x - y$ **51.** $\frac{c^2 - 2c + 4}{c}$ **53.** $\frac{xy}{x - y}$
55. $\frac{a^2 - 1}{a^2 + 1}$ **57.** $\frac{3(x - 1)^2(x + 2)}{(x - 3)(x + 3)(-x + 10)}$ **59.** $\frac{1 + a}{1 - a}$
61. $\frac{b + a}{b - a}$ **63.** Discussion and Writing **65.** $2x + h$
67. $3x^2 + 3xh + h^2$ **69.** x^5 **71.** $\frac{(n + 1)(n + 2)(n + 3)}{2 \cdot 3}$
73. $\frac{x^3 + 2x^2 + 11x + 20}{2(x + 1)(2 + x)}$