Algebra and Trigonometry in-class worksheet, Section R.4

Text excerpt:

Terms with Common Factors

When a polynomial is to be factored, we should always look first to factor out a factor that is common to all the terms using the distributive property. We usually look for the constant common factor with the largest absolute value and for variables with the largest exponent common to all the terms. In this sense, we factor out the "largest" common factor.

Factor out a common factor

1. $2x - 10$

3. $3x^4 - 9x^2$

5. $4a^2 - 12a + 16$

7. $a(b - 2) + c(b - 2)$

Text excerpt:

Factoring by Grouping

In some polynomials, pairs of terms have a common binomial factor that can be removed in a process called **factoring by grouping**.

Factor by Grouping

11. $y^3 - y^2 + 3y - 3$

17. $x^3 - x^2 - 5x + 5$

9. $x^3 + 3x^2 + 6x + 18$

15. $a^3 - 3a^2 - 2a + 6$

Text excerpt:

Trinomials of the Type $ax^2 + bx + c, a \neq 1$

We consider two methods for factoring trinomials of the type $ax^2 + bx + c$, $a \neq 1$.

The FOIL Method

We first consider the **FOIL method** for factoring trinomials of the type $ax^2 + bx + c$, $a \neq 1$.

however, finding such an answer involves trial and error. We use the following method.

To factor trinomials of the type $ax^2 + bx + c$, $a \neq 1$, using the **FOIL method**:

1. Factor out the largest common factor.
2. Find two First terms whose product is ax^2:

$$(\quad x + \quad)(\quad x + \quad) = ax^2 + bx + c.$$
F

3. Find two Last terms whose product is c:

$$(\quad x + \quad)(\quad x + \quad) = ax^2 + bx + c.$$
FOIL

4. Repeat steps (2) and (3) until a combination is found for which the sum of the Outside and Inside products is bx:

$$(\quad x + \quad)(\quad x + \quad) = ax^2 + bx + c.$$
O I FOIL

Factor the trinomial

21. $x^2 + 8x + 12$

23. $t^2 + 8t + 15$

27. $2n^2 - 20n - 48$

Some considerably harder trinomials to factor by the Grouping Method:

To factor $ax^2 + bx + c, a \neq 1$, using the **grouping method**:

1. Factor out the largest common factor.
2. Multiply the leading coefficient a and the constant c.
3. Try to factor the product ac so that the sum of the factors is b. That is, find integers p and q such that $pq = ac$ and $p + q = b$.
4. Split the middle term. That is, write it as a sum using the factors found in step (3).
5. Factor by grouping.

33. $12x^2 + 11x + 2$

39. $6a^2 - 29ab + 28b^2$

Special Factorizations

Factoring ***Difference of Squares***

$$A^2 - B^2 = (A + B)(A - B)$$

43. $m^2 - 4$

45. $9x^2 - 25$

47. $6x^2 - 6y^2$

Factoring ***Squares of Binomials***

$$A^2 + 2AB + B^2 = (A + B)^2;$$
$$A^2 - 2AB + B^2 = (A - B)^2.$$

53. $y^2 - 6y + 9$

55. $4z^2 + 12z + 9$

57. $1 - 8x + 16x^2$

59. $a^3 + 24a^2 + 144a$

Factoring ***sum or difference of cubes***

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$
$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

63. $x^3 + 8$

65. $m^3 - 1$

67. $2y^3 - 128$

69. $3a^5 - 24a^2$

Factoring... at the outer limit of what could ever reasonably be asked of a student with pencil & paper

99. $3z^3 - 24$

101. $16a^7b + 54ab^7$

103. $y^3 - 3y^2 - 4y + 12$

105. $x^3 - x^2 + x - 1$

107. $5m^4 - 20$

109. $2x^3 + 6x^2 - 8x - 24$

111. $4c^2 - 4cd + d^2$

Solutions

Exercise Set R.4

1. $2(x-5)$ **3.** $3x^2(x^2-3)$ **5.** $4(a^2-3a+4)$
7. $(b-2)(a+c)$ **9.** $(x+3)(x^2+6)$
11. $(y-1)(y^2+3)$ **13.** $12(2x-3)(x^2+3)$
15. $(a-3)(a^2-2)$ **17.** $(x-1)(x^2-5)$
19. $(p+2)(p+4)$ **21.** $(x+2)(x+6)$
23. $(t+3)(t+5)$ **25.** $(x+3y)(x-9y)$
27. $2(n-12)(n+2)$ **29.** $(y^2+3)(y^2-7)$
31. $(2n-7)(n+8)$ **33.** $(3x+2)(4x+1)$
35. $(4x+3)(x+3)$ **37.** $(2y-3)(y+2)$
39. $(3a-4b)(2a-7b)$ **41.** $4(3a-4)(a+1)$
43. $(m+2)(m-2)$ **45.** $(3x+5)(3x-5)$
47. $6(x+y)(x-y)$ **49.** $4x(y^2+z)(y^2-z)$
51. $7p(q^2+y^2)(q+y)(q-y)$ **53.** $(y-3)^2$
55. $(2z+3)^2$ **57.** $(1-4x)^2$ **59.** $a(a+12)^2$
61. $4(p-q)^2$ **63.** $(x+2)(x^2-2x+4)$
65. $(m-1)(m^2+m+1)$ **67.** $2(y-4)(y^2+4y+16)$
69. $3a^2(a-2)(a^2+2a+4)$ **71.** $(t^2+1)(t^4-t^2+1)$
73. $3ab(6a-5b)$ **75.** $(x-4)(x^2+5)$
77. $8(x+2)(x-2)$ **79.** Prime **81.** $(m+3n)(m-3n)$
83. $(x+4)(x+5)$ **85.** $(y-5)(y-1)$
87. $(2a+1)(a+4)$ **89.** $(3x-1)(2x+3)$
91. $(y-9)^2$ **93.** $(3z-4)^2$ **95.** $(xy-7)^2$
97. $4a(x+7)(x-2)$ **99.** $3(z-2)(z^2+2z+4)$
101. $2ab(2a^2+3b^2)(4a^4-6a^2b^2+9b^4)$
103. $(y-3)(y+2)(y-2)$ **105.** $(x-1)(x^2+1)$
107. $5(m^2+2)(m^2-2)$ **109.** $2(x+3)(x+2)(x-2)$
111. $(2c-d)^2$ **113.** $(m^3+10)(m^3-2)$
115. $p(1-4p)(1+4p+16p^2)$