

Text excerpt:

Algebraic expressions like $3ab^3 - 8$ and $5x^4y^2 - 3x^3y^8 + 7xy^2 + 6$ are **polynomials in several variables**. The **degree of a term** is the sum of the exponents of the variables in that term. The **degree of a polynomial** is the degree of the term of highest degree.

Determine the unique terms and their degree for these polynomials

1. $-5y^4 + 3y^3 + 7y^2 - y - 4$

3. $3a^4b - 7a^3b^3 + 5ab - 2$

4. $6p^3q^2 - p^2q^4 - 3pq^2 + 5$

Text excerpt:

If two terms of an expression have the same variables raised to the same powers, they are called **like terms**, or **similar terms**. We can **combine**, or **collect, like terms** using the distributive property. For example, $3y^2$ and $5y^2$ are like terms and

$$\begin{aligned}3y^2 + 5y^2 &= (3 + 5)y^2 \\ &= 8y^2.\end{aligned}$$

We add or subtract polynomials by combining like terms.

Perform the following polynomial additions

5. $(5x^2y - 2xy^2 + 3xy - 5) +$
 $(-2x^2y - 3xy^2 + 4xy + 7)$

6. $(6x^2y - 3xy^2 + 5xy - 3) +$
 $(-4x^2y - 4xy^2 + 3xy + 8)$

7. $(2x + 3y + z - 7) + (4x - 2y - z + 8) +$
 $(-3x + y - 2z - 4)$

9. $(3x^2 - 2x - x^3 + 2) - (5x^2 - 8x - x^3 + 4)$

11. $(x^4 - 3x^2 + 4x) - (3x^3 + x^2 - 5x + 3)$

Text excerpt:

Multiplication of polynomials is based on the distributive property—for example,

$$\begin{aligned}(x + 4)(x + 3) &= x(x + 3) + 4(x + 3) && \text{Using the distributive property} \\ &= x^2 + 3x + 4x + 12 && \text{Using the distributive property} \\ & && \text{two more times} \\ &= x^2 + 7x + 12. && \text{Combining like terms}\end{aligned}$$

In general, to multiply two polynomials, we multiply each term of one by each term of the other and add the products.

Perform the following polynomial multiplications

13. $(a - b)(2a^3 - ab + 3b^2)$

14. $(n + 1)(n^2 - 6n - 4)$

39. $(2x + 3y + 4)(2x + 3y - 4)$

Text excerpt:

We can find the product of two binomials by multiplying the **F**irst terms, then the **O**uter terms, then the **I**nner terms, then the **L**ast terms. Then we combine like terms, if possible. This procedure is sometimes called **FOIL**.

Special Products of Binomials

$$(A + B)^2 = A^2 + 2AB + B^2 \quad \text{Square of a sum}$$

$$(A - B)^2 = A^2 - 2AB + B^2 \quad \text{Square of a difference}$$

$$(A + B)(A - B) = A^2 - B^2 \quad \text{Product of a sum and a difference}$$

Perform the following (special) polynomial multiplications (ie binomial multiplication)

15. $(x + 5)(x - 3)$

16. $(y - 4)(y + 1)$

17. $(x + 6)(x + 4)$

18. $(n - 5)(n - 8)$

27. $(5x - 3)^2$

28. $(3x - 2)^2$

29. $(2x + 3y)^2$

33. $(a + 3)(a - 3)$

35. $(2x - 5)(2x + 5)$

37. $(3x - 2y)(3x + 2y)$

Rough and Tumble problems:

$$41. (x + 1)(x - 1)(x^2 + 1)$$

$$45. (a^n + b^n)(a^n - b^n)$$

$$49. (x - 1)(x^2 + x + 1)(x^3 + 1)$$

$$51. (x^{a-b})^{a+b}$$

$$53. (a + b + c)^2$$