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Algebra in-class worksheet, Section R.2

Integer Exponents, Scientific Notation, and Order of Operations

For any positive integer n ,

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

where a is the **base** and n is the **exponent**.

For any nonzero real number a and any integer m ,

$$a^0 = 1 \quad \text{and} \quad a^{-m} = \frac{1}{a^m}.$$

For any nonzero numbers a and b and any integers m and n ,

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}.$$

(A factor can be moved to the other side of the fraction bar if the sign of the exponent is changed.)

Properties of Exponents

For any real numbers a and b and any integers m and n , assuming 0 is not raised to a nonpositive power:

$$a^m \cdot a^n = a^{m+n} \quad \text{Product rule}$$

$$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0) \quad \text{Quotient rule}$$

$$(a^m)^n = a^{mn} \quad \text{Power rule}$$

$$(ab)^m = a^m b^m \quad \text{Raising a product to a power}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0) \quad \text{Raising a quotient to a power}$$

Scientific Notation

Scientific notation for a number is an expression of the type

$$N \times 10^m,$$

where $1 \leq N < 10$, N is in decimal notation, and m is an integer.

Rules for Order of Operations

1. Do all calculations within grouping symbols before operations outside. When nested grouping symbols are present, work from the inside out.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

Simplify.

1. 18^0

3. $x^9 \cdot x^0$

5. $5^8 \cdot 5^{-6}$

7. $m^{-5} \cdot m^5$

9. $y^3 \cdot y^{-7}$

11. $7^3 \cdot 7^{-5} \cdot 7$

13. $2x^3 \cdot 3x^2$

15. $(-3a^{-5})(5a^{-7})$

17. $(5a^2b)(3a^{-3}b^4)$

2. $(-\frac{4}{3})^0$

4. $a^0 \cdot a^4$

6. $6^2 \cdot 6^{-7}$

8. $n^9 \cdot n^{-9}$

10. $b^{-4} \cdot b^{12}$

12. $3^6 \cdot 3^{-5} \cdot 3^4$

14. $3y^4 \cdot 4y^3$

16. $(-6b^{-4})(2b^{-7})$

18. $(4xy^2)(3x^{-4}y^5)$

19. $(6x^{-3}y^5)(-7x^2y^{-9})$

21. $(2x)^3(3x)^2$

23. $(-2n)^3(5n)^2$

25. $\frac{b^{40}}{b^{37}}$

27. $\frac{x^{-5}}{x^{16}}$

29. $\frac{x^2y^{-2}}{x^{-1}y}$

31. $\frac{32x^{-4}y^3}{4x^{-5}y^8}$

20. $(8ab^7)(-7a^{-5}b^2)$

22. $(4y)^2(3y)^3$

24. $(2x)^5(3x)^2$

26. $\frac{a^{39}}{a^{32}}$

28. $\frac{y^{-24}}{y^{-21}}$

30. $\frac{x^3y^{-3}}{x^{-1}y^2}$

32. $\frac{20a^5b^{-2}}{5a^7b^{-3}}$

33. $(2ab^2)^3$

34. $(4xy^3)^2$

35. $(-2x^3)^5$

36. $(-3x^2)^4$

37. $(-5c^{-1}d^{-2})^{-2}$

38. $(-4x^{-5}z^{-2})^{-3}$

39. $(3m^4)^3(2m^{-5})^4$

40. $(4n^{-1})^2(2n^3)^3$

41. $\left(\frac{2x^{-3}y^7}{z^{-1}}\right)^3$

42. $\left(\frac{3x^5y^{-8}}{z^{-2}}\right)^4$

43. $\left(\frac{24a^{10}b^{-8}c^7}{12a^6b^{-3}c^5}\right)^{-5}$

44. $\left(\frac{125p^{12}q^{-14}r^{22}}{25p^8q^6r^{-15}}\right)^{-4}$

Convert to scientific notation.

45. 405,000

46. 1,670,000

47. 0.00000039

48. 0.00092

49. 234,600,000,000

50. 8,904,000,000

51. 0.00104

52. 0.00000000514

53. One cubic inch is approximately equal to
0.000016 m³.

54. The United States government collected
\$1,137,000,000,000 in individual income taxes in a
recent year (*Source*: U.S. Internal Revenue Service).

Convert to decimal notation.

55. 8.3×10^{-5}

56. 4.1×10^6

57. 2.07×10^7

58. 3.15×10^{-6}

59. 3.496×10^{10}

60. 8.409×10^{11}

61. 5.41×10^{-8}

62. 6.27×10^{-10}

63. The amount of solid waste generated in the United States in a recent year was 2.319×10^8 tons (*Source: Franklin Associates, Ltd.*).

64. The mass of a proton is about 1.67×10^{-24} g.

Compute. Write the answer using scientific notation.

65. $(3.1 \times 10^5)(4.5 \times 10^{-3})$

66. $(9.1 \times 10^{-17})(8.2 \times 10^3)$

67. $(2.6 \times 10^{-18})(8.5 \times 10^7)$

68. $(6.4 \times 10^{12})(3.7 \times 10^{-5})$

69. $\frac{6.4 \times 10^{-7}}{8.0 \times 10^6}$

70. $\frac{1.1 \times 10^{-40}}{2.0 \times 10^{-71}}$

71. $\frac{1.8 \times 10^{-3}}{7.2 \times 10^{-9}}$

72. $\frac{1.3 \times 10^4}{5.2 \times 10^{10}}$

Solve. Write the answer using scientific notation.

73. *Distance to Pluto.* The distance from Earth to the sun is defined as 1 **astronomical unit**, or AU. It is about 93 million miles. The average distance from Earth to the planet Pluto is 39 AUs. Find this distance in miles.
74. *Parsecs.* One **parsec** is about 3.26 light-years and 1 light-year is about 5.88×10^{12} mi. Find the number of miles in 1 parsec.
75. *Nanowires.* A **nanometer** is 0.000000001 m. Scientists have developed optical nanowires to transmit light waves short distances. A nanowire with a diameter of 360 nanometers has been used in experiments on the transmission of light (*Source: New York Times*, January 29, 2004). Find the diameter of such a wire in meters.
76. *iTunes.* In the first quarter of 2004, Apple Computer was selling 2.7 million songs per week on iTunes, its online music service (*Source: Apple Computer*). At \$0.99 per song, what is the revenue during a 13-week period?
77. *Chesapeake Bay Bridge-Tunnel.* The 17.6-mile-long Chesapeake Bay Bridge-Tunnel was completed in 1964. Construction costs were \$210 million. Find the average cost per mile.
78. *Personal Space in Hong Kong.* The area of Hong Kong is 412 square miles. It is estimated that the population of Hong Kong will be 9,600,000 in 2050. Find the number of square miles of land per person in 2050.
79. *Nuclear Disintegration.* One gram of radium produces 37 billion disintegrations per second. How many disintegrations are produced in 1 hr?
80. *Length of Earth's Orbit.* The average distance from the earth to the sun is 93 million mi. About how far does the earth travel in a yearly orbit? (Assume a circular orbit.)

Calculate.

$$81. 3 \cdot 2 - 4 \cdot 2^2 + 6(3 - 1)$$

$$82. 3[(2 + 4 \cdot 2^2) - 6(3 - 1)]$$

$$83. 16 \div 4 \cdot 4 \div 2 \cdot 256$$

$$84. 2^6 \cdot 2^{-3} \div 2^{10} \div 2^{-8}$$

$$85. \frac{4(8 - 6)^2 - 4 \cdot 3 + 2 \cdot 8}{3^1 + 19^0}$$

$$86. \frac{[4(8 - 6)^2 + 4](3 - 2 \cdot 8)}{2^2(2^3 + 5)}$$

Simplify. Assume that all exponents are integers, all denominators are nonzero, and zero is not raised to a nonpositive power.

$$97. (x^t \cdot x^{3t})^2$$

$$98. (x^y \cdot x^{-y})^3$$

$$99. (t^{a+x} \cdot t^{x-a})^4$$

$$100. (m^{x-b} \cdot n^{x+b})^x (m^b n^{-b})^x$$

$$101. \left[\frac{(3x^a y^b)^3}{(-3x^a y^b)^2} \right]^2$$

$$102. \left[\left(\frac{x^r}{y^t} \right)^2 \left(\frac{x^{2r}}{y^{4t}} \right)^{-2} \right]^{-3}$$

Exercise Set R.2

1. 1 3. x^9 5. 5^2 , or 25 7. 1 9. y^{-4} , or $\frac{1}{y^4}$
11. 7^{-1} , or $\frac{1}{7}$ 13. $6x^5$ 15. $-15a^{-12}$, or $-\frac{15}{a^{12}}$
17. $15a^{-1}b^5$, or $\frac{15b^5}{a}$ 19. $-42x^{-1}y^{-4}$, or $-\frac{42}{xy^4}$
21. $72x^5$ 23. $-200n^5$ 25. b^3 27. x^{-21} , or $\frac{1}{x^{21}}$
29. x^3y^{-3} , or $\frac{x^3}{y^3}$ 31. $8xy^{-5}$, or $\frac{8x}{y^5}$ 33. $8a^3b^6$
35. $-32x^{15}$ 37. $\frac{c^2d^4}{25}$ 39. $432m^{-8}$, or $\frac{432}{m^8}$
41. $\frac{8x^{-9}y^{21}}{z^{-3}}$, or $\frac{8y^{21}z^3}{x^9}$ 43. $2^{-5}a^{-20}b^{25}c^{-10}$, or $\frac{b^{25}}{32a^{20}c^{10}}$
45. 4.05×10^5 47. 3.9×10^{-7} 49. 2.346×10^{11}
51. 1.04×10^{-3} 53. 1.6×10^{-5} 55. 0.000083
57. 20,700,000 59. 34,960,000,000 61. 0.0000000541
63. 231,900,000 65. 1.395×10^3 67. 2.21×10^{-10}
69. 8×10^{-14} 71. 2.5×10^5 73. 3.627×10^9 mi
75. 3.6×10^{-7} m 77. $\$1.19 \times 10^7$
79. 1.332×10^{14} disintegrations 81. 2 83. 2048 85. 5
87. \$2883.67 89. \$8763.54 91. Discussion and Writing
93. \$170,797.30 95. \$309.79 97. x^{8t} 99. t^{8x}
101. $9x^{2a}y^{2b}$