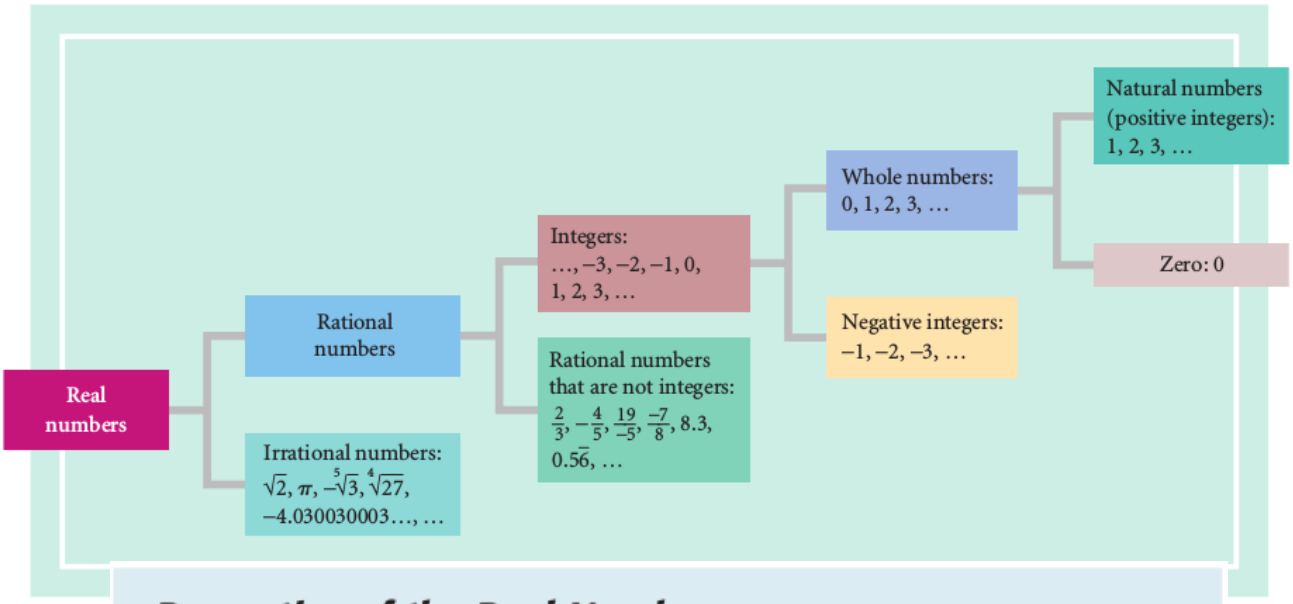
02-Dec-2015
Algebra in-class worksheet, Section R.1

Chapter R Basic Concepts of Algebra
R.1 The Real-Number System
R.2 Integer Exponents, Scientific Notation, and Order of Operations
R.3 Addition, Subtraction, and Multiplication of Polynomials
R.4 Factoring
R.5 Rational Expressions
R.6 Radical Notation and Rational Exponents
R.7 The Basics of Equation Solving

R.1: The Real-Number System



Properties of the Real Numbers

For any real numbers a, b, and c:

$a + b = b + a$ and $ab = ba$	Commutative properties of addition and multiplication
$a + (b + c) = (a + b) + c$ and $a(bc) = (ab)c$	Associative properties of addition and multiplication
$a + 0 = 0 + a = a$	Additive identity property
$-a + a = a + (-a) = 0$	Additive inverse property
$a \cdot 1 = 1 \cdot a = a$	Multiplicative identity property
$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \quad (a \neq 0)$	Multiplicative inverse property
$a(b + c) = ab + ac$	Distributive property

Intervals: Types, Notation, and Graphs

Type	Interval Notation	Set Notation	Graph
Open	(a, b)	$\{x \mid a < x < b\}$	
Closed	$[a, b]$	$\{x \mid a \le x \le b\}$	
Half-open	$[a, b)$	$\{x \mid a \le x < b\}$	
Half-open	$(a, b]$	$\{x \mid a < x \le b\}$	
Open	(a, ∞)	$\{x \mid x > a\}$	
Half-open	$[a, \infty)$	$\{x \mid x \ge a\}$	
Open	$(-\infty, b)$	$\{x \mid x < b\}$	
Half-open	$(-\infty, b]$	$\{x \mid x \le b\}$	

Absolute Value

For any real number a,

$$|a| = \begin{cases} a, & \text{if } a \ge 0, \\ -a, & \text{if } a < 0. \end{cases}$$

When a is nonnegative, the absolute value of a is a. When a is negative, the absolute value of a is the opposite, or additive inverse, of a. Thus, $|a|$ is never negative; that is, for any real number a, $|a| \ge 0$.

Distance Between Two Points on the Number Line

For any real numbers a and b, the **distance between a and b** is $|a - b|$, or equivalently, $|b - a|$.

In Exercises 1–10, consider the numbers $-12, \sqrt{7}, 5.\overline{3}, -\frac{7}{3}, \sqrt[3]{8}, 0, 5.242242224\ldots, -\sqrt{14}, \sqrt[5]{5}, -1.96, 9, 4\frac{2}{3}, \sqrt{25}, \sqrt[3]{4}, \frac{5}{7}$.

1. Which are whole numbers?

2. Which are integers?

3. Which are irrational numbers?

4. Which are natural numbers?

5. Which are rational numbers?

6. Which are real numbers?

7. Which are rational numbers but not integers?

8. Which are integers but not whole numbers?

9. Which are integers but not natural numbers?

10. Which are real numbers but not integers?

Write interval notation. Then graph the interval.

11. $\{x|-3 \leq x \leq 3\}$

12. $\{x|-4 < x < 4\}$

13. $\{x|-4 \leq x < -1\}$

14. $\{x|1 < x \leq 6\}$

15. $\{x|x \leq -2\}$

16. $\{x|x > -5\}$

17. $\{x|x > 3.8\}$

18. $\{x|x \geq \sqrt{3}\}$

19. $\{x|7 < x\}$

20. $\{x|-3 > x\}$

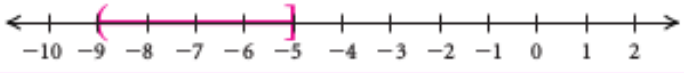
Write interval notation for the graph.

21.

22.

23.

24.



25.

26.

27.

28.

In Exercises 29–46, the following notation is used: $\mathbb{N}$ = *the set of natural numbers,* $\mathbb{W}$ = *the set of whole numbers,* $\mathbb{Z}$ = *the set of integers,* $\mathbb{Q}$ = *the set of rational numbers,* $\mathbb{I}$ = *the set of irrational numbers, and* $\mathbb{R}$ = *the set of real numbers. Classify the statement as true or false.*

29. $6 \in \mathbb{N}$

30. $0 \notin \mathbb{N}$

31. $3.2 \in \mathbb{Z}$

32. $-10.\overline{1} \in \mathbb{R}$

33. $-\frac{11}{5} \in \mathbb{Q}$

34. $-\sqrt{6} \in \mathbb{Q}$

35. $\sqrt{11} \notin \mathbb{R}$

36. $-1 \in \mathbb{W}$

37. $24 \notin \mathbb{W}$

38. $1 \in \mathbb{Z}$

39. $1.089 \notin \mathbb{I}$

40. $\mathbb{N} \subseteq \mathbb{W}$

41. $\mathbb{W} \subseteq \mathbb{Z}$

42. $\mathbb{Z} \subseteq \mathbb{N}$

43. $\mathbb{Q} \subseteq \mathbb{R}$

44. $\mathbb{Z} \subseteq \mathbb{Q}$

45. $\mathbb{R} \subseteq \mathbb{Z}$

46. $\mathbb{Q} \subseteq \mathbb{I}$

Name the property illustrated by the sentence.

47. $6 \cdot x = x \cdot 6$

48. $3 + (x + y) = (3 + x) + y$

49. $-3 \cdot 1 = -3$

50. $x + 4 = 4 + x$

51. $5(ab) = (5a)b$

52. $4(y - z) = 4y - 4z$

53. $2(a + b) = (a + b)2$ **54.** $-7 + 7 = 0$

55. $-6(m + n) = -6(n + m)$

56. $t + 0 = t$

57. $8 \cdot \frac{1}{8} = 1$

58. $9x + 9y = 9(x + y)$

Simplify.

59. $|-7.1|$ **60.** $|-86.2|$

61. $|347|$ **62.** $|-54|$

63. $\left|-\sqrt{97}\right|$ **64.** $\left|\frac{12}{19}\right|$

65. $|0|$ **66.** $|15|$

67. $\left|\frac{5}{4}\right|$ **68.** $\left|-\sqrt{3}\right|$

Find the distance between the given pair of points on the number line.

69. $-5,\ 6$ **70.** $-2.5,\ 0$

71. $-8,\ -2$ **72.** $\frac{15}{8},\ \frac{23}{12}$

73. $6.7,\ 12.1$ **74.** $-14,\ -3$

75. $-\frac{3}{4},\ \frac{15}{8}$ **76.** $-3.4,\ 10.2$

77. $-7,\ 0$ **78.** $3,\ 19$

Collaborative Discussion and Writing

To the student and the instructor: The Collaborative Discussion and Writing exercises are meant to be answered with one or more sentences. These exercises can also be discussed and answered collaboratively by the entire class or by small groups. Because of their open-ended nature, the answers to these exercises do not appear at the back of the book. They are denoted by the words "Discussion and Writing."

79. How would you convince a classmate that division is not associative?

80. Under what circumstances is $\sqrt{a}$ a rational number?

Synthesis

To the student and the instructor: The Synthesis exercises found at the end of every exercise set challenge students to combine concepts or skills studied in that section or in preceding parts of the text.

Between any two (different) real numbers there are many other real numbers. Find each of the following. Answers may vary.

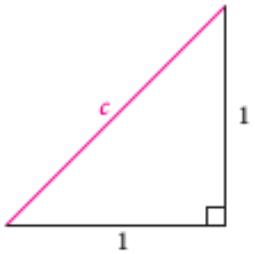
81. An irrational number between 0.124 and 0.125

82. A rational number between $-\sqrt{2.01}$ and $-\sqrt{2}$

83. A rational number between $-\frac{1}{101}$ and $-\frac{1}{100}$

84. An irrational number between $\sqrt{5.99}$ and $\sqrt{6}$

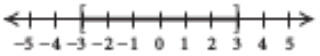
85. The hypotenuse of an isosceles right triangle with legs of length 1 unit can be used to "measure" a value for $\sqrt{2}$ by using the Pythagorean theorem, as shown.

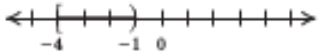


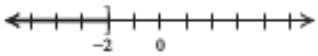
$$c^2 = 1^2 + 1^2$$
$$c^2 = 2$$
$$c = \sqrt{2}$$

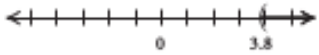
Draw a right triangle that could be used to "measure" $\sqrt{10}$ units.

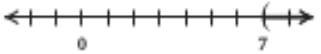
Exercise Set R.1

1. $\sqrt[3]{8}, 0, 9, \sqrt{25}$
3. $\sqrt{7}, 5.242242224\ldots, -\sqrt{14}, \sqrt[5]{5}, \sqrt[3]{4}$
5. $-12, 5.\overline{3}, -\frac{7}{3}, \sqrt[3]{8}, 0, -1.96, 9, 4\frac{2}{3}, \sqrt{25}, \frac{5}{7}$
7. $5.\overline{3}, -\frac{7}{3}, -1.96, 4\frac{2}{3}, \frac{5}{7}$ **9.** $-12, 0$
11. $[-3, 3]$; 

13. $[-4, -1)$; 

15. $(-\infty, -2]$; 

17. $(3.8, \infty)$; 

19. $(7, \infty)$; 

21. $(0, 5)$
23. $[-9, -4)$ **25.** $[x, x + h]$ **27.** (p, ∞) **29.** True
31. False **33.** True **35.** False **37.** False **39.** True
41. True **43.** True **45.** False **47.** Commutative property of multiplication
49. Multiplicative identity property
51. Associative property of multiplication
53. Commutative property of multiplication
55. Commutative property of addition
57. Multiplicative inverse property **59.** 7.1 **61.** 347
63. $\sqrt{97}$ **65.** 0 **67.** $\frac{5}{4}$ **69.** 11 **71.** 6 **73.** 5.4
75. $\frac{21}{8}$ **77.** 7 **79.** Discussion and Writing

81. Answers may vary; 0.124124412444 . . .
83. Answers may vary; -0.00999 **85.**