Algebra in-class worksheet TO BE HANDED IN AND GRADED!
Chapter 4.4 & 4.5: Properties of Logarithmic functions; Solving equations

Name:_____
Date: _____

Factoids to be used in solving problems:

The Product Rule

For any positive numbers M and N and any logarithmic base a,

$$\log_a MN = \log_a M + \log_a N.$$

(The logarithm of a product is the sum of the logarithms of the factors.)

The Power Rule

For any positive number M, any logarithmic base a, and any real number p,

$$\log_a M^p = p \log_a M.$$

(The logarithm of a power of M is the exponent times the logarithm of M.)

The Quotient Rule

For any positive numbers M and N, and any logarithmic base a,

$$\log_a \frac{M}{N} = \log_a M - \log_a N.$$

(The logarithm of a quotient is the logarithm of the numerator minus the logarithm of the denominator.)

The Logarithm of a Base to a Power

For any base a and any real number x,

$$\log_a a^x = x.$$

(The logarithm, base a, of a to a power is the power.)

A Base to a Logarithmic Power

For any base a and any positive real number x,

$$a^{\log_a x} = x.$$

(The number a raised to the power $\log_a x$ is x.)

Common Errors

$\log_a MN \neq (\log_a M)(\log_a N)$ — The logarithm of a product is *not* the product of the logarithms.

$\log_a (M + N) \neq \log_a M + \log_a N$ — The logarithm of a sum is *not* the sum of the logarithms.

$\log_a \dfrac{M}{N} \neq \dfrac{\log_a M}{\log_a N}$ — The logarithm of a quotient is *not* the quotient of the logarithms.

$(\log_a M)^p \neq p \log_a M$ — The power of a logarithm is *not* the exponent times the logarithm.

Summary of the Properties of Logarithms

The Product Rule:	$\log_a MN = \log_a M + \log_a N$
The Power Rule:	$\log_a M^p = p \log_a M$
The Quotient Rule:	$\log_a \dfrac{M}{N} = \log_a M - \log_a N$
The Change-of-Base Formula:	$\log_b M = \dfrac{\log_a M}{\log_a b}$
Other Properties:	$\log_a a = 1, \qquad \log_a 1 = 0,$
	$\log_a a^x = x, \qquad a^{\log_a x} = x$

Do odd # problems

Express as a sum of logarithms.

1. $\log_3(81 \cdot 27)$
2. $\log_2(8 \cdot 64)$
3. $\log_5(5 \cdot 125)$
4. $\log_4(64 \cdot 32)$
5. $\log_t 8Y$
6. $\log 0.2x$
7. $\ln xy$
8. $\ln ab$

Express as a product.

9. $\log_b t^3$
10. $\log_a x^4$
11. $\log y^8$
12. $\ln y^5$
13. $\log_c K^{-6}$
14. $\log_b Q^{-8}$
15. $\ln \sqrt[3]{4}$
16. $\ln \sqrt{a}$

Express as a difference of logarithms.

17. $\log_t \dfrac{M}{8}$

18. $\log_a \dfrac{76}{13}$

19. $\log \dfrac{x}{y}$

20. $\ln \dfrac{a}{b}$

21. $\ln \dfrac{r}{s}$

22. $\log_b \dfrac{3}{w}$

Express in terms of sums and differences of logarithms.

23. $\log_a 6xy^5z^4$

24. $\log_a x^3y^2z$

25. $\log_b \dfrac{p^2q^5}{m^4b^9}$

26. $\log_b \dfrac{x^2y}{b^3}$

27. $\ln \dfrac{2}{3x^3y}$

28. $\log \dfrac{5a}{4b^2}$

29. $\log \sqrt{r^3t}$

30. $\ln \sqrt[3]{5x^5}$

31. $\log_a \sqrt{\dfrac{x^6}{p^5q^8}}$

32. $\log_c \sqrt[3]{\dfrac{y^3z^2}{x^4}}$

33. $\log_a \sqrt[4]{\dfrac{m^8n^{12}}{a^3b^5}}$

34. $\log_a \sqrt{\dfrac{a^6b^8}{a^2b^5}}$

Express as a single logarithm and, if possible, simplify.

35. $\log_a 75 + \log_a 2$

36. $\log 0.01 + \log 1000$

37. $\log 10{,}000 - \log 100$

38. $\ln 54 - \ln 6$

39. $\frac{1}{2}\log n + 3\log m$

40. $\frac{1}{2}\log a - \log 2$

41. $\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x$

42. $\frac{2}{5}\log_a x - \frac{1}{3}\log_a y$

43. $\ln x^2 - 2\ln\sqrt{x}$

44. $\ln 2x + 3(\ln x - \ln y)$

45. $\ln(x^2 - 4) - \ln(x + 2)$

46. $\log(x^3 - 8) - \log(x - 2)$

47. $\log(x^2 - 5x - 14) - \log(x^2 - 4)$

48. $\log_a \dfrac{a}{\sqrt{x}} - \log_a \sqrt{ax}$

49. $\ln x - 3[\ln(x - 5) + \ln(x + 5)]$

50. $\frac{2}{3}[\ln(x^2 - 9) - \ln(x + 3)] + \ln(x + y)$

51. $\frac{3}{2}\ln 4x^6 - \frac{4}{5}\ln 2y^{10}$

52. $120\left(\ln\sqrt[5]{x^3} + \ln\sqrt[3]{y^2} - \ln\sqrt[4]{16z^5}\right)$

4.5

Exercise Set

Solve the exponential equation algebraically. Then check using a graphing calculator.

1. $3^x = 81$

2. $2^x = 32$

3. $2^{2x} = 8$

4. $3^{7x} = 27$

5. $2^x = 33$

6. $2^x = 40$

7. $5^{4x-7} = 125$

8. $4^{3x-5} = 16$

9. $27 = 3^{5x} \cdot 9^{x^2}$

10. $3^{x^2+4x} = \frac{1}{27}$

11. $84^x = 70$

12. $28^x = 10^{-3x}$

13. $e^{-c} = 5^{2c}$

14. $15^x = 30$

15. $e^t = 1000$

16. $e^{-t} = 0.04$

17. $e^{-0.03t} = 0.08$

18. $1000e^{0.09t} = 5000$

19. $3^x = 2^{x-1}$

20. $5^{x+2} = 4^{1-x}$

21. $(3.9)^x = 48$

22. $250 - (1.87)^x = 0$

23. $e^x + e^{-x} = 5$

24. $e^x - 6e^{-x} = 1$

25. $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$

26. $\dfrac{5^x - 5^{-x}}{5^x + 5^{-x}} = 8$

Solve the logarithmic equation algebraically. Then check using a graphing calculator.

27. $\log_5 x = 4$

28. $\log_2 x = -3$

29. $\log x = -4$

30. $\log x = 1$

31. $\ln x = 1$

32. $\ln x = -2$

33. $\log_2 (10 + 3x) = 5$

34. $\log_5 (8 - 7x) = 3$

35. $\log x + \log(x - 9) = 1$

36. $\log_2 (x + 1) + \log_2 (x - 1) = 3$

37. $\log_2 (x + 20) - \log_2 (x + 2) = \log_2 x$

38. $\log (x + 5) - \log (x - 3) = \log 2$

39. $\log_8 (x + 1) - \log_8 x = 2$

40. $\log x - \log(x + 3) = -1$

41. $\log x + \log(x + 4) = \log 12$

42. $\ln x - \ln (x - 4) = \ln 3$

43. $\log_4 (x + 3) + \log_4 (x - 3) = 2$

Exercise Set 4.4

1. $\log_3 81 + \log_3 27$ 3. $\log_5 5 + \log_5 125$
5. $\log_t 8 + \log_t Y$ 7. $\ln x + \ln y$ 9. $3\log_b t$
11. $8\log y$ 13. $-6\log_c K$ 15. $\frac{1}{3}\ln 4$
17. $\log_t M - \log_t 8$ 19. $\log x - \log y$ 21. $\ln r - \ln s$
23. $\log_a 6 + \log_a x + 5\log_a y + 4\log_a z$
25. $2\log_b p + 5\log_b q - 4\log_b m - 9$
27. $\ln 2 - \ln 3 - 3\ln x - \ln y$
29. $\frac{3}{2}\log r + \frac{1}{2}\log t$ 31. $3\log_a x - \frac{5}{2}\log_a p - 4\log_a q$
33. $2\log_a m + 3\log_a n - \frac{3}{4} - \frac{5}{4}\log_a b$ 35. $\log_a 150$
37. $\log 100 = 2$ 39. $\log m^3\sqrt{n}$

41. $\log_a x^{-5/2}y^4$, or $\log_a \dfrac{y^4}{x^{5/2}}$ 43. $\ln x$ 45. $\ln(x-2)$

47. $\log \dfrac{x-7}{x-2}$ 49. $\ln \dfrac{x}{(x^2-25)^3}$ 51. $\ln \dfrac{2^{11/5}x^9}{y^8}$

53. -0.74 55. 1.991 57. 0.356 59. 4.827
61. -1.792 63. 0.099 65. 3 67. $|x-4|$ 69. $4x$
71. w 73. $8t$ 75. $\frac{1}{2}$ 77. Discussion and Writing
79. [3.1] Quartic   80. [4.2] Exponential
81. [1.4] Linear (constant)   82. [4.2] Exponential
83. [3.5] Rational   84. [4.3] Logarithmic
85. [3.1] Cubic   86. [3.5] Rational   87. [1.4] Linear
88. [2.4] Quadratic   89. 4 91. $\log_a(x^3-y^3)$
93. $\frac{1}{2}\log_a(x-y) - \frac{1}{2}\log_a(x+y)$ 95. 7 97. True
99. True   101. True   103. -2 105. 3

107. $e^{-xy} = \dfrac{a}{b}$

109. $\log_a \left(\dfrac{x+\sqrt{x^2-5}}{5} \cdot \dfrac{x-\sqrt{x^2-5}}{x-\sqrt{x^2-5}} \right)$

$$= \log_a \dfrac{5}{5\left(x-\sqrt{x^2-5}\right)}$$
$$= -\log_a\left(x-\sqrt{x^2-5}\right)$$

Exercise Set 4.5

1. 4 3. $\frac{3}{2}$ 5. 5.044 7. $\frac{5}{2}$ 9. $-3,\frac{1}{2}$ 11. 0.959
13. 0 15. 6.908 17. 84.191 19. -1.710 21. 2.844
23. $-1.567, 1.567$ 25. 0.347 27. 625 29. 0.0001
31. e 33. $\frac{22}{3}$ 35. 10 37. 4 39. $\frac{1}{63}$ 41. 2
43. 5 45. $\frac{21}{8}$ 47. $\frac{8}{7}$ 49. 0.367 51. 0.621
53. -1.532 55. 7.062 57. 2.444 59. $(4.093, 0.786)$