Algebra in-class worksheet TO BE HANDED IN AND GRADED!
Chapter 4.1 & 4.2: Exponential and Logarithmic functions

Name:______________________
Date: _______________________

Copy down word-for-word the text's definition of an ***Exponential Function*** (pp 361)

Compare and contrast: $y_1 = f_1(x) = 2^x$ versus $y_2 = f_2(x) = x^2$
Evaluate these two functions at each value of x and plot on the same graph

x	f1(x)	f2(x)
-3		
-2		
-1		
0		
1		
2		
3		

As x gets larger than 3, which function becomes larger? (evaluate at x=4, x=10, x=20)

The number *e:*

$$e = 2.7182818284\ldots$$

e is an irrational number that occurs in natural phenomena.
e has comparable social-scientific significance to the number π
(For instance, if the February had at least 71 days, I'm sure people would celebrate "*e* day" and have contests to see who could spout off from memory the most digits at 8:28 in the morning.)

Calculator exercises: (check your work against the answers on page 368

EXAMPLE 5 Find each value of e^x, to four decimal places, using the $\boxed{e^x}$ key on a calculator.

a) e^3 **b)** $e^{-0.23}$ **c)** e^0 **d)** e^1

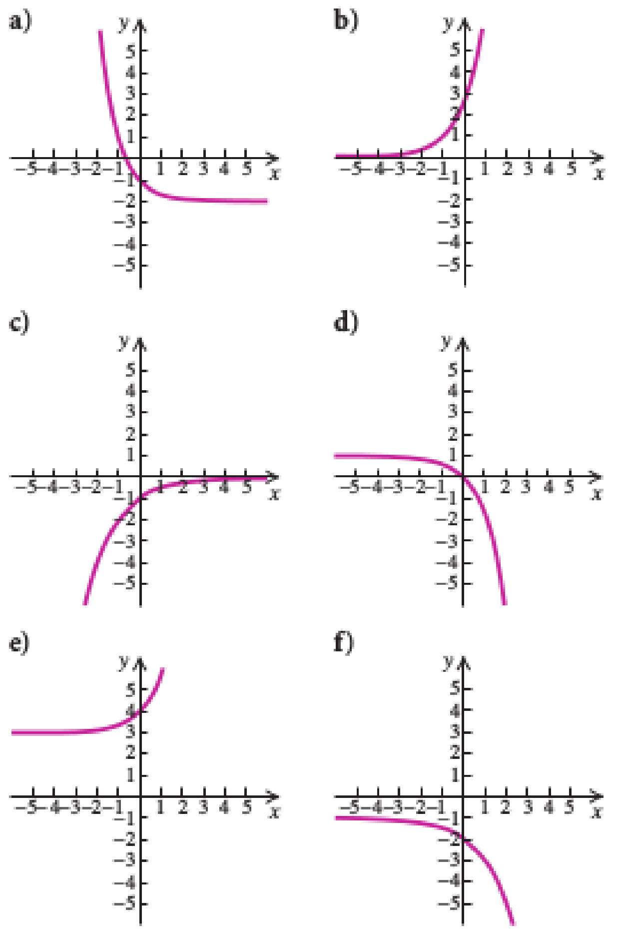
Textbook problems on exponential functions:

In Exercises 5–10, match the function with one of the graphs (a)–(f), which follow.

5. $f(x) = -2^x - 1$ **6.** $f(x) = -(\frac{1}{2})^x$

7. $f(x) = e^x + 3$ **8.** $f(x) = e^{x+1}$

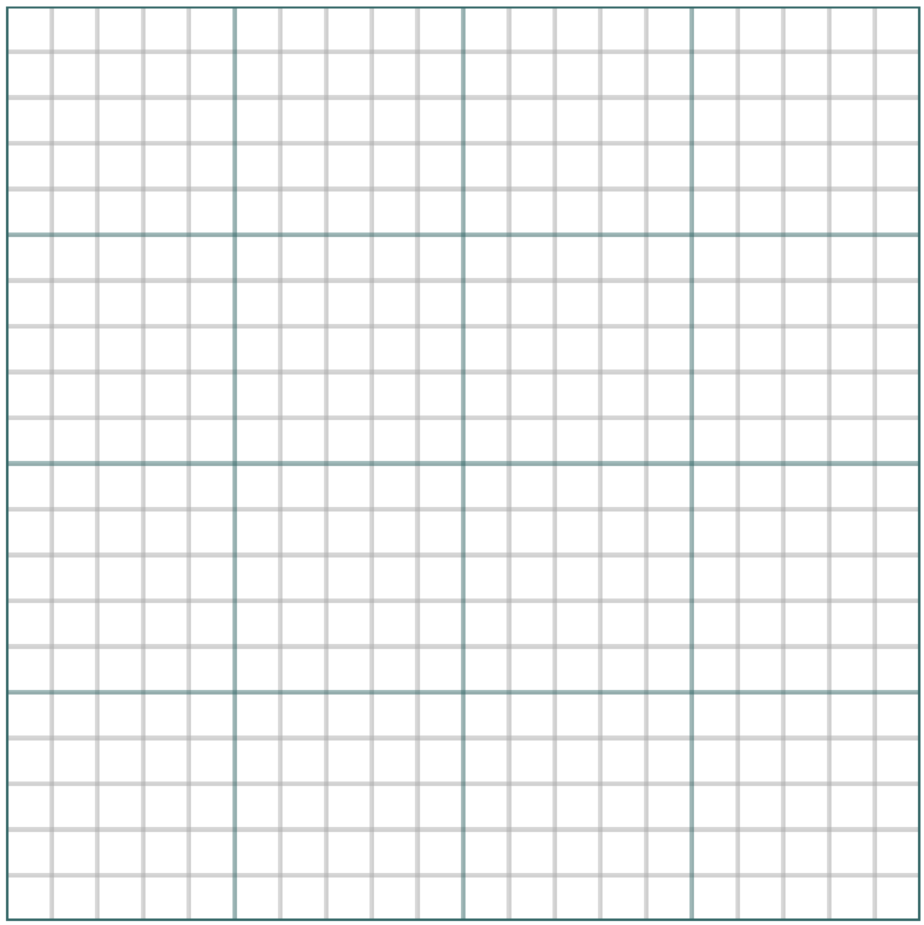
9. $f(x) = 3^{-x} - 2$ **10.** $f(x) = 1 - e^x$



Recycling Aluminum Cans. It is estimated that two thirds of all aluminum cans distributed will be recycled each year (*Source*: Alcoa Corporation). A beverage company distributes 350,000 cans. The number still in use after time t, in years, is given by the exponential function

$$N(t) = 350{,}000\left(\frac{2}{3}\right)^t.$$

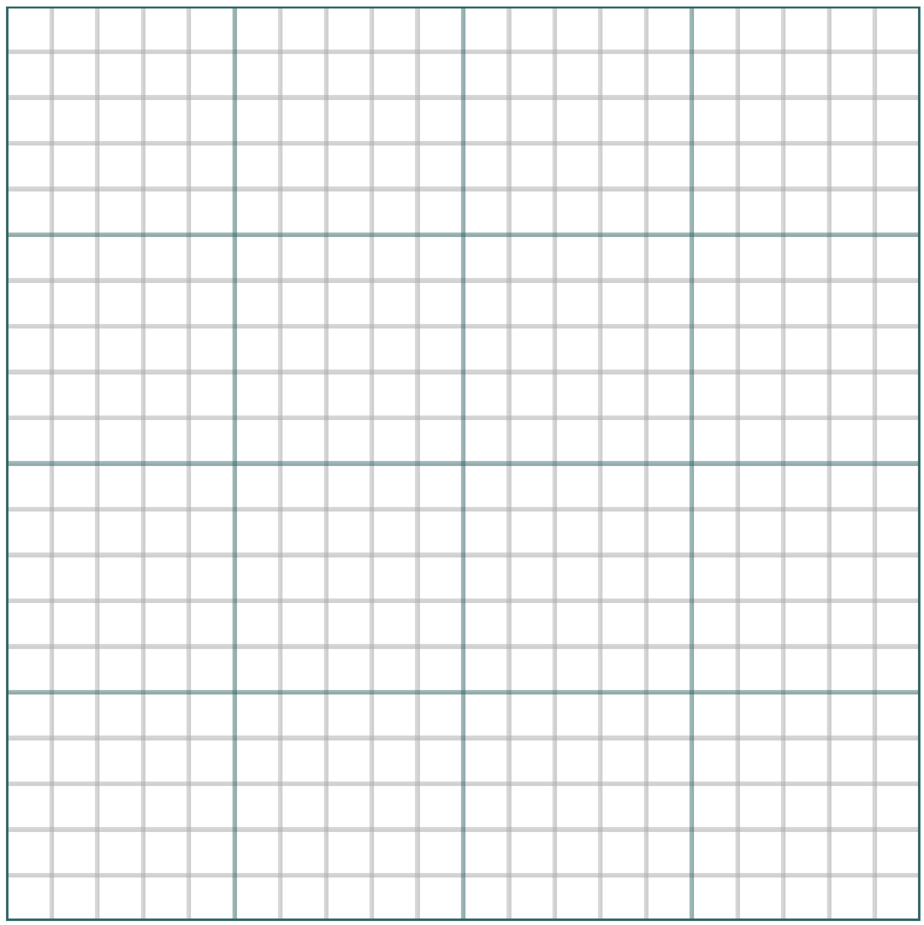
a) How many cans are still in use after 0 yr? 1 yr? 4 yr? 10 yr?
b) Graph the function.
c) After how long will 100,000 cans still be in use?



61. *Salvage Value.* A top-quality phone-fax copying machine is purchased for $1800. Its value each year is about 80% of the value of the preceding year. After t years, its value, in dollars, is given by the exponential function

$$V(t) = 1800(0.8)^t.$$

a) Graph the function.
b) Find the value of the machine after 0 yr, 1 yr, 2 yr, 5 yr, and 10 yr.
c) The company decides to replace the machine when its value has declined to $500. After how long will the machine be replaced?

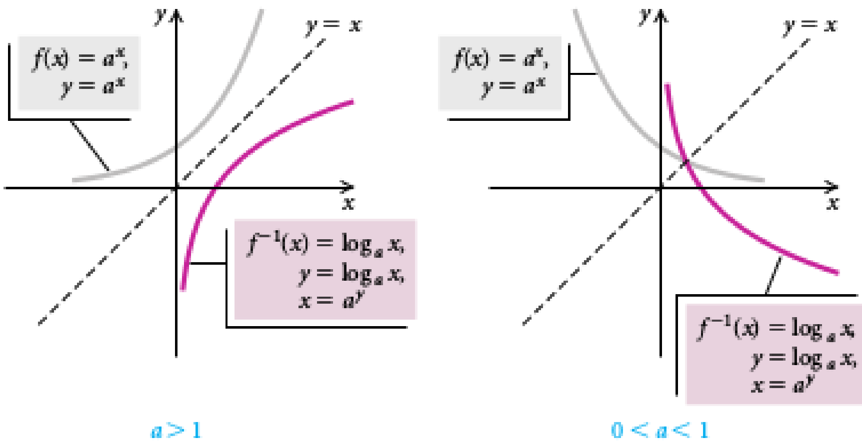


Logarithmic Functions

Logarithmic Function, Base a

We define $y = \log_a x$ as that number y such that $x = a^y$, where $x > 0$ and a is a positive constant other than 1.

Let's look at the graphs of $f(x) = a^x$ and $f^{-1}(x) = \log_a x$ for $a > 1$ and $0 < a < 1$.



Note that the graphs of $f(x)$ and $f^{-1}(x)$ are reflections of each other across the line $y = x$.

Logarithmic Function, Base 2

"$\log_2 x$," read "the logarithm, base 2, of x," means "the power to which we raise 2 to get x."

$$\log x \quad \text{means} \quad \log_{10} x.$$

$$\ln x \quad \text{means} \quad \log_e x.$$

Quick exercises (check your work against examples on page 378)

EXAMPLE 3 Convert each of the following to a logarithmic equation.

a) $16 = 2^x$ **b)** $10^{-3} = 0.001$ **c)** $e^t = 70$

Additional Facts about Logarithms:

$\log_a 1 = 0$ and $\log_a a = 1$, for any logarithmic base a.

The Change-of-Base Formula

For any logarithmic bases a and b, and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

We will prove this result in the next section.

EXAMPLE 7 Find $\log_5 8$ using common logarithms.

Solution First, we let $a = 10$, $b = 5$, and $M = 8$. Then we substitute into the change-of-base formula:

$$\log_5 8 = \frac{\log_{10} 8}{\log_{10} 5} \quad \text{Substituting}$$

$$\approx 1.2920. \quad \text{Using a calculator}$$

Since $\log_5 8$ is the power to which we raise 5 to get 8, we would expect this power to be greater than 1 ($5^1 = 5$) and less than 2 ($5^2 = 25$), so the result is reasonable. The check is shown in the window at left.

Do each of these odd-numbered problems. Expect to be quizzed on the even-numbered problems

Find each of the following. Do not use a calculator.

9. $\log_2 16$	**10.** $\log_3 9$
11. $\log_5 125$	**12.** $\log_2 64$
13. $\log 0.001$	**14.** $\log 100$
15. $\log_2 \frac{1}{4}$	**16.** $\log_8 2$
17. $\ln 1$	**18.** $\ln e$
19. $\log 10$	**20.** $\log 1$
21. $\log_5 5^4$	**22.** $\log \sqrt{10}$
23. $\log_3 \sqrt[4]{3}$	**24.** $\log 10^{8/5}$
25. $\log 10^{-7}$	**26.** $\log_5 1$
27. $\log_{49} 7$	**28.** $\log_3 3^{-2}$
29. $\ln e^{3/4}$	**30.** $\log_2 \sqrt{2}$
31. $\log_4 1$	**32.** $\ln e^{-5}$
33. $\ln \sqrt{e}$	**34.** $\log_{64} 4$

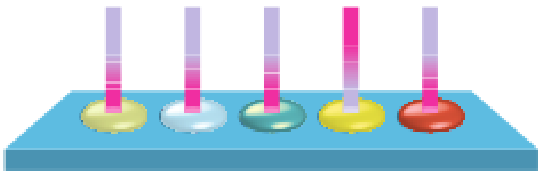
Find the logarithm using common logarithms and the change-of-base formula.

69. $\log_4 100$	**70.** $\log_3 20$
71. $\log_{100} 0.3$	**72.** $\log_{\pi} 100$
73. $\log_{200} 50$	**74.** $\log_{5.3} 1700$

94. *pH of Substances in Chemistry.* In chemistry, the pH of a substance is defined as

$$pH = -\log[H^+],$$

where H^+ is the hydrogen ion concentration, in moles per liter. Find the pH of each substance.



Litmus paper is used to test pH.

	Substance	Hydrogen Ion Concentration
a)	Pineapple juice	1.6×10^{-4}
b)	Hair rinse	0.0013
c)	Mouthwash	6.3×10^{-7}
d)	Eggs	1.6×10^{-8}
e)	Tomatoes	6.3×10^{-5}

95. Find the hydrogen ion concentration of each substance, given the pH (see Exercise 94). Express the answer in scientific notation.

	Substance	pH
a)	Tap water	7
b)	Rainwater	5.4
c)	Orange juice	3.2
d)	Wine	4.8