Algebra Ch1-7
Symmetry and Transformations

In a nutshell:

Algebraic Tests of Symmetry

x-axis: If replacing y with $-y$ produces an equivalent equation, then the graph is *symmetric with respect to the x-axis.*

y-axis: If replacing x with $-x$ produces an equivalent equation, then the graph is *symmetric with respect to the y-axis.*

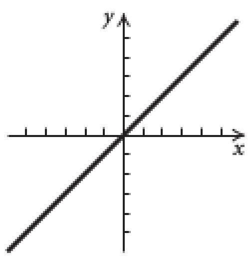
Origin: If replacing x with $-x$ and y with $-y$ produces an equivalent equation, then the graph is *symmetric with respect to the origin.*

Even and Odd Functions

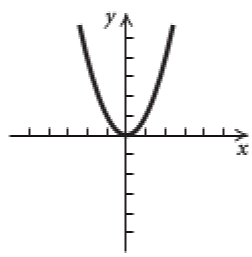
If the graph of a function f is symmetric with respect to the y-axis, we say that it is an **even function**. That is, for each x in the domain of f, $f(x) = f(-x)$.

If the graph of a function f is symmetric with respect to the origin, we say that it is an **odd function**. That is, for each x in the domain of f, $f(-x) = -f(x)$.

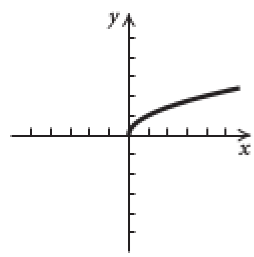
Identity function:
$y = x$



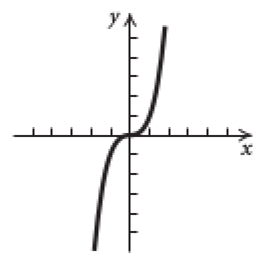
Squaring function:
$y = x^2$



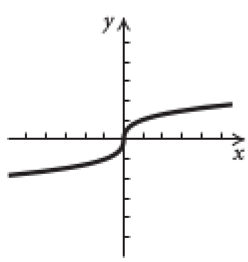
Square root function:
$y = \sqrt{x}$



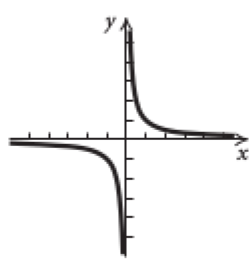
Cubing function:
$y = x^3$



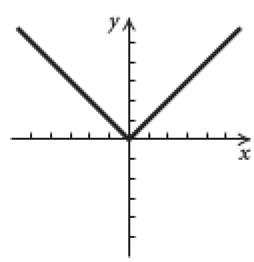
Cube root function:
$y = \sqrt[3]{x}$



Reciprocal function:
$y = \frac{1}{x}$



Absolute-value function:
$y = |x|$

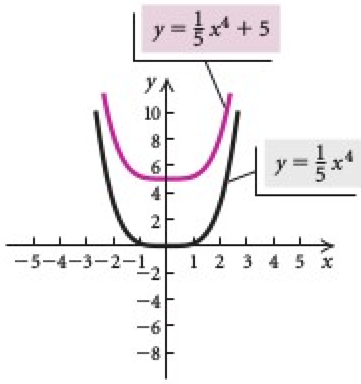


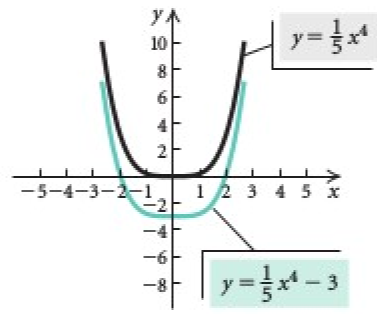
Vertical Translation

For $b > 0$,

the graph of $y = f(x) + b$ is the graph of $y = f(x)$ shifted *up* b units;

the graph of $y = f(x) - b$ is the graph of $y = f(x)$ shifted *down* b units.



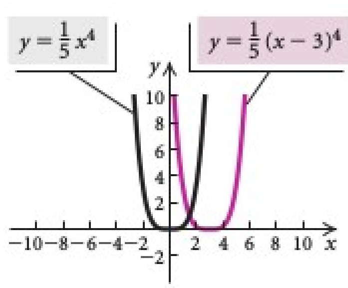


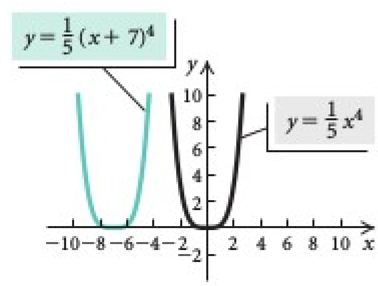
Horizontal Translation

For $d > 0$:

the graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted *right* d units;

the graph of $y = f(x + d)$ is the graph of $y = f(x)$ shifted *left* d units.



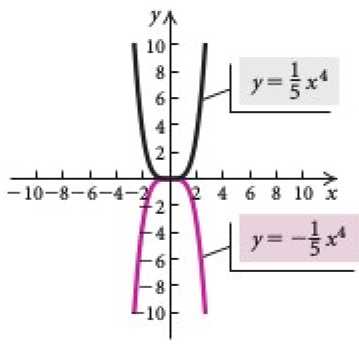


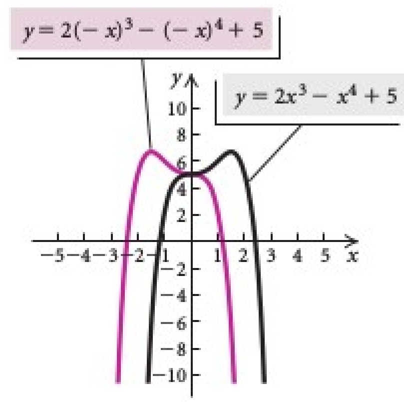
Reflections

The graph of $y = -f(x)$ is the **reflection** of the graph of $y = f(x)$ across the x-axis.

The graph of $y = f(-x)$ is the **reflection** of the graph of $y = f(x)$ across the y-axis.

If a point (x, y) is on the graph of $y = f(x)$, then $(x, -y)$ is on the graph of $y = -f(x)$, and $(-x, y)$ is on the graph of $y = f(-x)$.





Vertical Stretching and Shrinking

The graph of $y = af(x)$ can be obtained from the graph of $y = f(x)$ by

stretching vertically for $|a| > 1$, or
shrinking vertically for $0 < |a| < 1$.

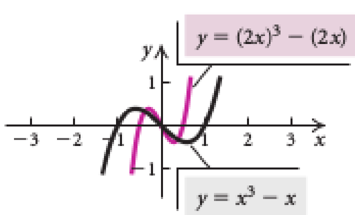
For $a < 0$, the graph is also reflected across the x-axis. (The y-coordinates of the graph of $y = af(x)$ can be obtained by multiplying the y-coordinates of $y = f(x)$ by a.)

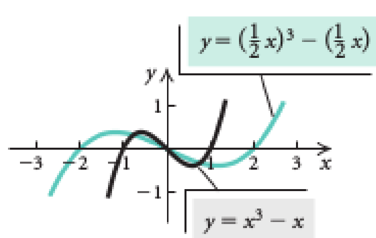
Horizontal Stretching and Shrinking

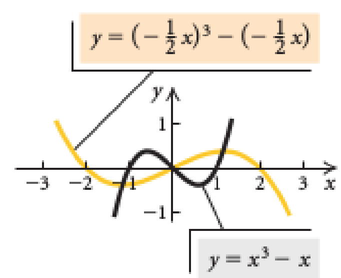
The graph of $y = f(cx)$ can be obtained from the graph of $y = f(x)$ by

shrinking horizontally for $|c| > 1$, or
stretching horizontally for $0 < |c| < 1$.

For $c < 0$, the graph is also reflected across the y-axis. (The x-coordinates of the graph of $y = f(cx)$ can be obtained by dividing the x-coordinates of the graph of $y = f(x)$ by c.)

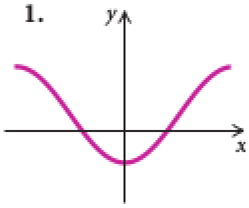






Determine visually whether the graph is symmetric with respect to the x-axis, the y-axis, and the origin.

1.



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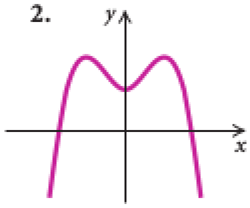

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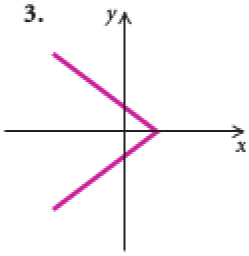

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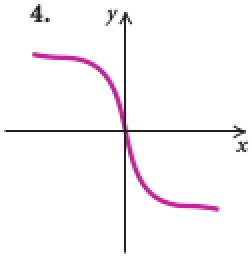

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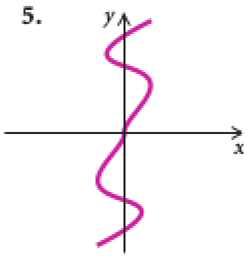

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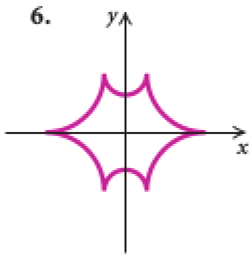


First, graph the equation and determine visually whether it is symmetric with respect to the x-axis, the y-axis, and the origin. Then verify your assertion algebraically.

7. $y = |x| - 2$

8. $y = |x + 5|$

9. $5y = 4x + 5$

10. $2x - 5 = 3y$

11. $5y = 2x^2 - 3$

12. $x^2 + 4 = 3y$

13. $y = \dfrac{1}{x}$

14. $y = -\dfrac{4}{x}$

Test algebraically whether the graph is symmetric with respect to the x-axis, the y-axis, and the origin. Then check your work graphically, if possible, using a graphing calculator.

15. $5x - 5y = 0$

16. $6x + 7y = 0$

17. $3x^2 - 2y^2 = 3$

18. $5y = 7x^2 - 2x$

19. $y = |2x|$

20. $y^3 = 2x^2$

21. $2x^4 + 3 = y^2$

22. $2y^2 = 5x^2 + 12$

23. $3y^3 = 4x^3 + 2$

24. $3x = |y|$

25. $xy = 12$

26. $xy - x^2 = 3$

Find the point that is symmetric to the given point with respect to the x-axis; the y-axis; the origin.

27. $(-5, 6)$

28. $\left(\frac{7}{2}, 0\right)$

29. $(-10, -7)$

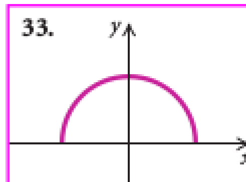
30. $\left(1, \frac{3}{8}\right)$

31. $(0, -4)$

32. $(8, -3)$

Determine visually whether the function is even, odd, or neither even nor odd.

33.



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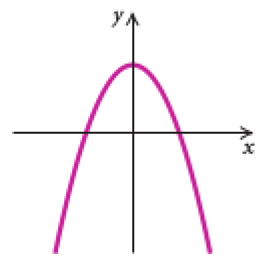

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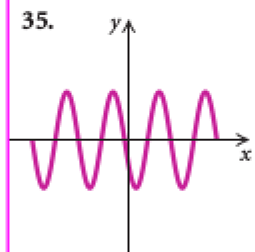

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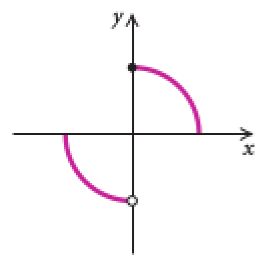

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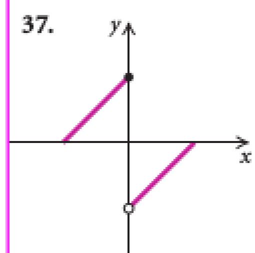

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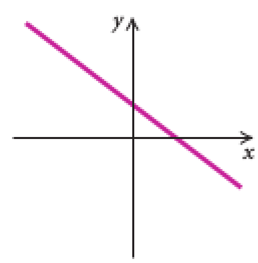


Test algebraically whether the function is even, odd, or neither even nor odd. Then check your work graphically, where possible, using a graphing calculator.

39. $f(x) = -3x^3 + 2x$

40. $f(x) = 7x^3 + 4x - 2$

41. $f(x) = 5x^2 + 2x^4 - 1$

42. $f(x) = x + \dfrac{1}{x}$

43. $f(x) = x^{17}$

44. $f(x) = \sqrt[3]{x}$

45. $f(x) = x - |x|$

46. $f(x) = \dfrac{1}{x^2}$

47. $f(x) = 8$

48. $f(x) = \sqrt{x^2 + 1}$

Describe how the graph of the function can be obtained from one of the basic graphs on page 153. Then graph the function by hand or with a graphing calculator.

49. $f(x) = (x - 3)^2$

50. $g(x) = x^2 + \frac{1}{2}$

51. $g(x) = x - 3$

52. $g(x) = -x - 2$

53. $h(x) = -\sqrt{x}$

54. $g(x) = \sqrt{x - 1}$

55. $h(x) = \dfrac{1}{x} + 4$

56. $g(x) = \dfrac{1}{x - 2}$

57. $h(x) = -3x + 3$

58. $f(x) = 2x + 1$

59. $h(x) = \frac{1}{2}|x| - 2$

60. $g(x) = -|x| + 2$

61. $g(x) = -(x - 2)^3$

62. $f(x) = (x + 1)^3$

63. $g(x) = (x + 1)^2 - 1$

64. $h(x) = -x^2 - 4$

65. $g(x) = \frac{1}{3}x^3 + 2$

66. $h(x) = (-x)^3$

67. $f(x) = \sqrt{x + 2}$

68. $f(x) = -\frac{1}{2}\sqrt{x - 1}$

69. $f(x) = \sqrt[3]{x} - 2$

70. $h(x) = \sqrt[3]{x + 1}$

Describe how the graph of the function can be obtained from one of the basic graphs on page 153.

71. $g(x) = |3x|$

72. $f(x) = \frac{1}{2}\sqrt[3]{x}$

73. $h(x) = \dfrac{2}{x}$

74. $f(x) = |x - 3| - 4$

75. $f(x) = 3\sqrt{x} - 5$

76. $f(x) = 5 - \dfrac{1}{x}$

77. $g(x) = \left|\frac{1}{3}x\right| - 4$

78. $f(x) = \frac{2}{3}x^3 - 4$

79. $f(x) = -\frac{1}{4}(x - 5)^2$

80. $f(x) = (-x)^3 - 5$

81. $f(x) = \dfrac{1}{x + 3} + 2$

82. $g(x) = \sqrt{-x} + 5$

83. $h(x) = -(x - 3)^2 + 5$

84. $f(x) = 3(x + 4)^2 - 3$

The point $(-12, 4)$ is on the graph of $y = f(x)$. Find the corresponding point on the graph of $y = g(x)$.

85. $g(x) = \frac{1}{2}f(x)$

86. $g(x) = f(x - 2)$

87. $g(x) = f(-x)$

88. $g(x) = f(4x)$

89. $g(x) = f(x) - 2$

90. $g(x) = f\left(\frac{1}{2}x\right)$

91. $g(x) = 4f(x)$

92. $g(x) = -f(x)$

Write an equation for a function that has a graph with the given characteristics. Check your answer using a graphing calculator.

93. The shape of $y = x^2$, but upside down and shifted right 8 units

94. The shape of $y = \sqrt{x}$, but shifted left 6 units and down 5 units

95. The shape of $y = |x|$, but shifted left 7 units and up 2 units

96. The shape of $y = x^3$, but upside down and shifted right 5 units

97. The shape of $y = 1/x$, but shrunk horizontally by a factor of 2 and shifted down 3 units

98. The shape of $y = x^2$, but shifted right 6 units and up 2 units

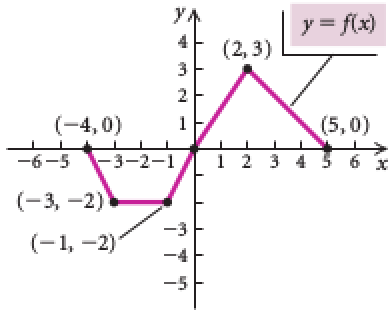
99. The shape of $y = x^2$, but upside down and shifted right 3 units and up 4 units

100. The shape of $y = |x|$, but stretched horizontally by a factor of 2 and shifted down 5 units

101. The shape of $y = \sqrt{x}$, but reflected across the y-axis and shifted left 2 units and down 1 unit

102. The shape of $y = 1/x$, but reflected across the x-axis and shifted up 1 unit

A graph of $y = f(x)$ follows. No formula for f is given. In Exercises 103–110, make a hand-drawn graph of the function.



103. $g(x) = -2f(x)$

104. $g(x) = \frac{1}{2}f(x)$

105. $g(x) = f\left(-\frac{1}{2}x\right)$

106. $g(x) = f(2x)$

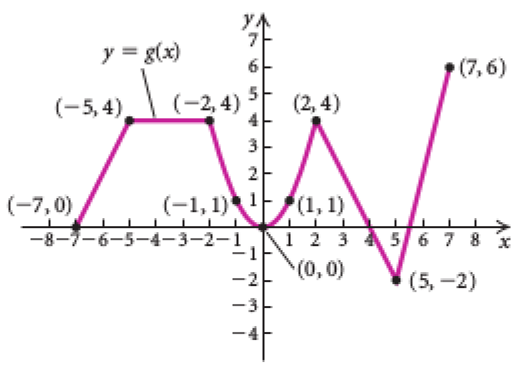
107. $g(x) = -\frac{1}{2}f(x-1) + 3$

108. $g(x) = -3f(x+1) - 4$

109. $g(x) = f(-x)$

110. $g(x) = -f(x)$

A graph of $y = g(x)$ follows. No formula for g is given. In Exercises 111–114, make a hand-drawn graph of the function.



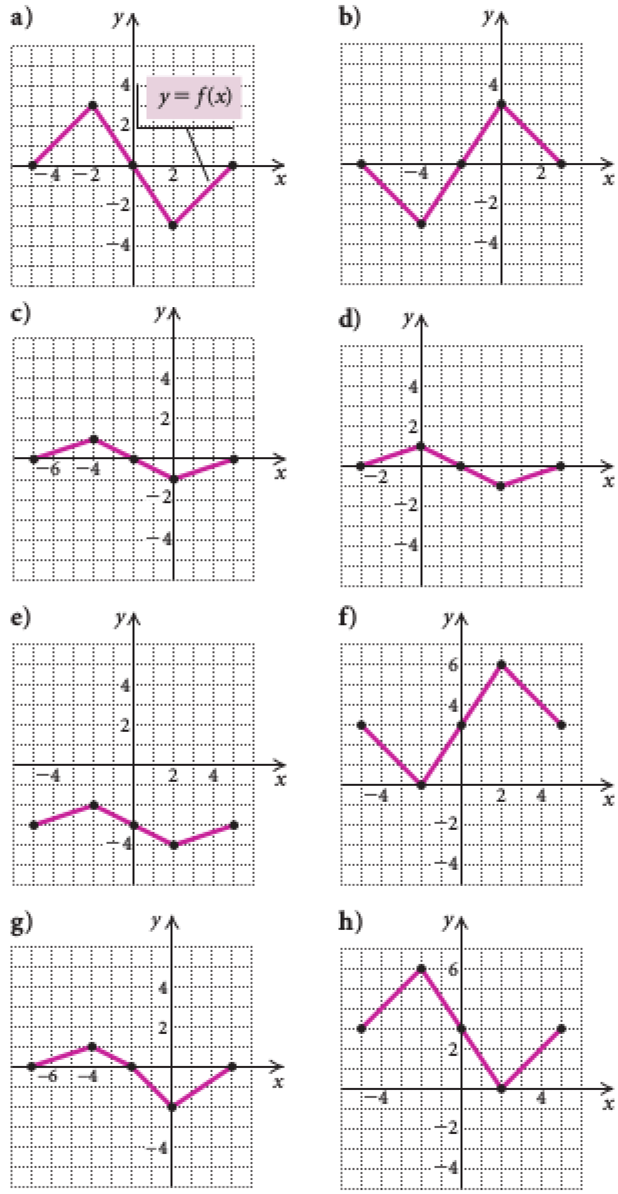
111. $h(x) = -g(x+2) + 1$

112. $h(x) = \frac{1}{2}g(-x)$

113. $h(x) = g(2x)$

114. $h(x) = 2g(x-1) - 3$

The graph of the function f is shown in figure (a). In Exercises 115–122, match the function g with one of the graphs (a)–(h), which follow. Some graphs may be used more than once and some may not be used.



115. $g(x) = f(-x) + 3$

116. $g(x) = f(x) + 3$

117. $g(x) = -f(x) + 3$

118. $g(x) = -f(-x)$

119. $g(x) = \frac{1}{3}f(x-2)$

120. $g(x) = \frac{1}{3}f(x) - 3$

121. $g(x) = \frac{1}{3}f(x+2)$

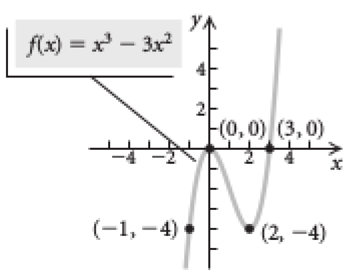
122. $g(x) = -f(x+2)$

For each pair of functions, determine algebraically if $g(x) = f(-x)$. Then, using the TABLE *feature on a graphing calculator, check your answers.*

123. $f(x) = 2x^4 - 35x^3 + 3x - 5,$
$g(x) = 2x^4 + 35x^3 - 3x - 5$

124. $f(x) = \frac{1}{4}x^4 + \frac{1}{5}x^3 - 81x^2 - 17,$
$g(x) = \frac{1}{4}x^4 + \frac{1}{5}x^3 + 81x^2 - 17$

A graph of the function $f(x) = x^3 - 3x^2$ is shown below. Exercises 125–128 show graphs of functions transformed from this one. Find a formula for each function.



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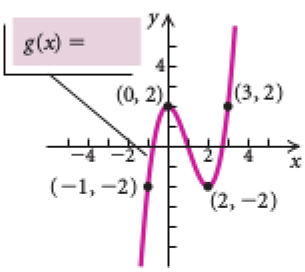

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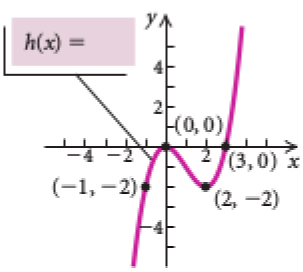

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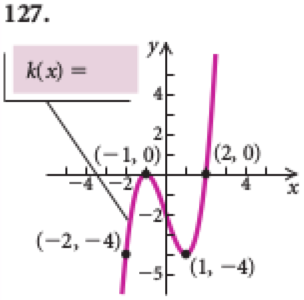

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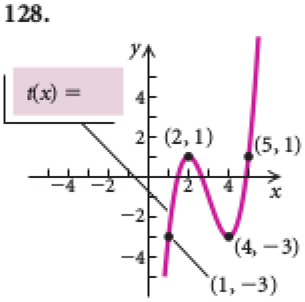


Collaborative Discussion and Writing

129. Consider the constant function $f(x) = 0$. Determine whether the graph of this function is symmetric with respect to the x-axis, the y-axis, and the origin. Determine whether this function is even or odd.

130. Describe conditions under which you would know whether a polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ is even or odd without using an algebraic procedure. Explain.

131. Explain in your own words why the graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ across the y-axis.

132. Without drawing the graph, describe what the graph of $f(x) = |x^2 - 9|$ looks like.

Skill Maintenance

133. Given $f(x) = 5x^2 - 7$, find each of the following.

a) $f(-3)$
b) $f(3)$
c) $f(a)$
d) $f(-a)$

134. Given $f(x) = 4x^3 - 5x$, find each of the following.

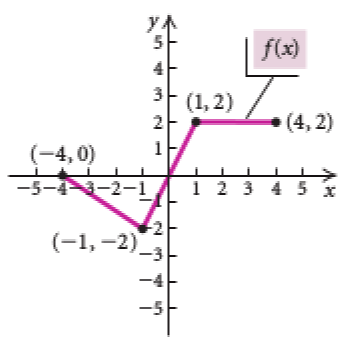
a) $f(2)$
b) $f(-2)$
c) $f(a)$
d) $f(-a)$

135. Write an equation of the line perpendicular to the graph of the line $8x - y = 10$ and containing the point $(-1, 1)$.

136. Find the slope and the y-intercept of the line with equation $2x - 9y + 1 = 0$.

Synthesis

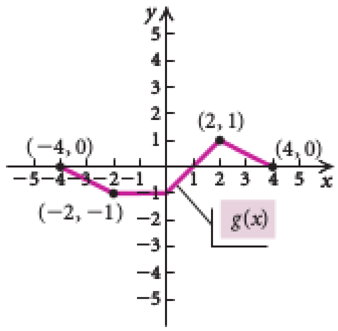
Use the graph of the function f shown below in Exercises 137 and 138.



137. Graph: $y = |f(x)|$.

138. Graph: $y = f(|x|)$.

Use the graph of the function g shown below in Exercises 139 and 140.



139. Graph: $y = |g(x)|$.

140. Graph: $y = g(|x|)$.

Graph each of the following using a graphing calculator. Before doing so, describe how the graph can be obtained from a more basic graph. Give the domain and the range of the function.

141. $f(x) = [\![x - \frac{1}{2}]\!]$

142. $f(x) = \left|\sqrt{x} - 1\right|$

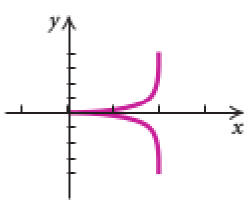
Determine whether the function is even, odd, or neither even nor odd.

143. $f(x) = x\sqrt{10 - x^2}$

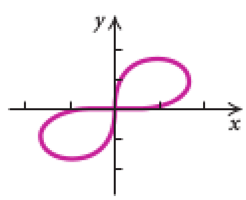
144. $f(x) = \dfrac{x^2 + 1}{x^3 - 1}$

Determine whether the graph is symmetric with respect to the x-axis, the y-axis, and the origin.

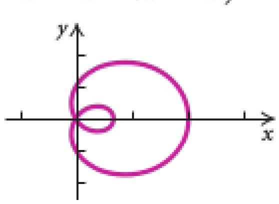
145. $x^3 = y^2(2 - x)$



146. $(x^2 + y^2)^2 = 2xy$



147. $y^2 + 4xy^2 - y^4 =$
$x^4 - 4x^3 + 3x^2 + 2x^2y^2$



148. The graph of $f(x) = |x|$ passes through the points $(-3, 3)$, $(0, 0)$, and $(3, 3)$. Transform this function to one whose graph passes through the points $(5, 1)$, $(8, 4)$, and $(11, 1)$.

149. If $(-1, 5)$ is a point on the graph of $y = f(x)$, find b such that $(2, b)$ is on the graph of $y = f(x - 3)$.

150. Find the zeros of $f(x) = 3x^5 - 20x^3$. Then without using a graphing calculator, state the zeros of $f(x - 3)$ and $f(x + 8)$.

151. If $(3, 4)$ is a point on the graph of $y = f(x)$, what point do you know is on the graph of $y = 2f(x)$? of $y = 2 + f(x)$? of $y = f(2x)$?

State whether each of the following is true or false.

152. The product of two odd functions is odd.

153. The sum of two even functions is even.

154. The product of an even function and an odd function is odd.

155. Show that if f is *any* function, then the function E defined by

$$E(x) = \frac{f(x) + f(-x)}{2}$$

is even.

156. Show that if f is *any* function, then the function O defined by

$$O(x) = \frac{f(x) - f(-x)}{2}$$

is odd.

157. Consider the functions E and O of Exercises 155 and 156.

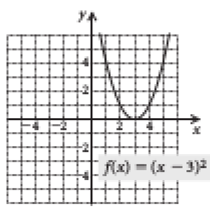
a) Show that $f(x) = E(x) + O(x)$. This means that every function can be expressed as the sum of an even and an odd function.

b) Let $f(x) = 4x^3 - 11x^2 + \sqrt{x} - 10$. Express f as a sum of an even function and an odd function.

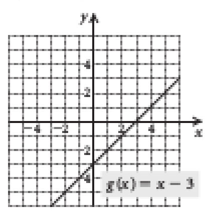
Exercise Set 1.7

1. x-axis, no; y-axis, yes; origin, no
3. x-axis, yes; y-axis, no; origin, no
5. x-axis, no; y-axis, no; origin, yes
7. x-axis, no; y-axis, yes; origin, no
9. x-axis, no; y-axis, no; origin, no
11. x-axis, no; y-axis, yes; origin, no
13. x-axis, no; y-axis, no; origin, yes
15. x-axis, no; y-axis, no; origin, yes
17. x-axis, yes; y-axis, yes; origin, yes
19. x-axis, no; y-axis, yes; origin, no
21. x-axis, yes; y-axis, yes; origin, yes
23. x-axis, no; y-axis, no; origin, no
25. x-axis, no; y-axis, no; origin, yes
27. x-axis: $(-5, -6)$; y-axis: $(5, 6)$; origin: $(5, -6)$
29. x-axis: $(-10, 7)$; y-axis: $(10, -7)$; origin: $(10, 7)$
31. x-axis: $(0, 4)$; y-axis: $(0, -4)$; origin: $(0, 4)$
33. Even **35.** Odd **37.** Neither **39.** Odd **41.** Even

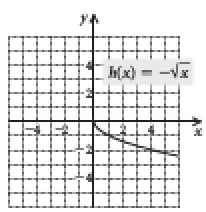
43. Odd **45.** Neither **47.** Even
49. Start with the graph of $f(x) = x^2$. Shift it right 3 units.



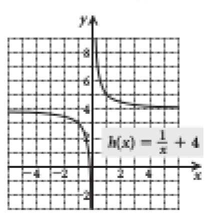
51. Start with the graph of $g(x) = x$. Shift it down 3 units.



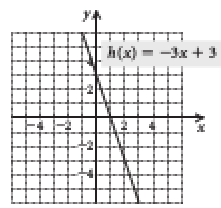
53. Start with the graph of $h(x) = \sqrt{x}$. Reflect it across the x-axis.



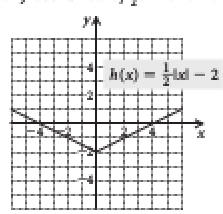
55. Start with the graph of $h(x) = \dfrac{1}{x}$. Shift it up 4 units.



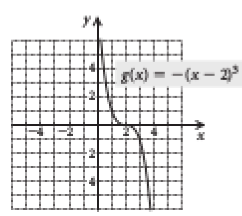
57. Start with the graph of $h(x) = x$. Stretch it vertically by multiplying each y-coordinate by 3. Then reflect it across the x-axis and shift it up 3 units.



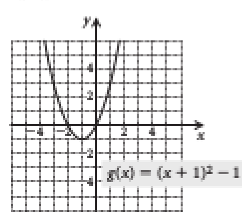
59. Start with the graph of $h(x) = |x|$. Shrink it vertically by multiplying each y-coordinate by $\frac{1}{2}$. Then shift it down 2 units.



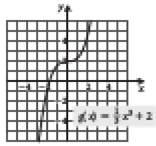
61. Start with the graph of $g(x) = x^3$. Shift it right 2 units. Then reflect it across the x-axis.



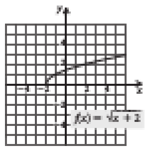
63. Start with the graph of $g(x) = x^2$. Shift it left 1 unit. Then shift it down 1 unit.



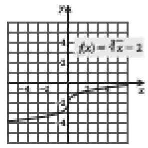
65. Start with the graph of $g(x) = x^3$. Shrink it vertically by multiplying each y-coordinate by $\frac{1}{2}$. Then shift it up 2 units.



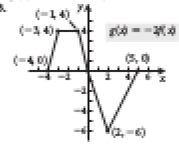
67. Start with the graph of $f(x) = \sqrt{x}$. Shift it left 2 units.



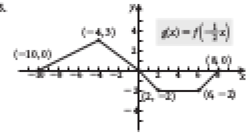
69. Start with the graph of $f(x) = \sqrt[3]{x}$. Shift it down 2 units.



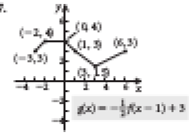
71. Start with the graph of $f(x) = |x|$. Shrink it horizontally by multiplying each x-coordinate by $\frac{1}{3}$ (or dividing each x-coordinate by 3).
73. Start with the graph of $h(x) = \dfrac{1}{x}$. Stretch it vertically by multiplying each y-coordinate by 2.
75. Start with the graph of $g(x) = \sqrt{x}$. Stretch it vertically by multiplying each y-coordinate by 3. Then shift it down 5 units.
77. Start with the graph of $f(x) = |x|$. Stretch it horizontally by multiplying each x-coordinate by 3. Then shift it down 4 units.
79. Start with the graph of $g(x) = x^2$. Shift it right 5 units, shrink it vertically by multiplying each y-coordinate by $\frac{1}{2}$, and reflect it across the x-axis.
81. Start with the graph of $g(x) = 1/x$. Shift it left 3 units, then up 2 units.
83. Start with the graph of $h(x) = x^2$. Shift it right 3 units. Then reflect it across the x-axis and shift it up 5 units.
85. $(-12, 2)$ 87. $(12, 4)$ 89. $(-12, 2)$ 91. $(-12, 16)$
93. $f(x) = -(x-8)^2$ 95. $f(x) = |x+7| + 2$
97. $f(x) = \dfrac{1}{2x} - 3$ 99. $f(x) = -(x-3)^2 + 4$
101. $f(x) = \sqrt{-(x+2)} - 1$
103.



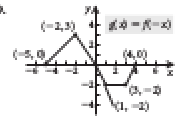
105.



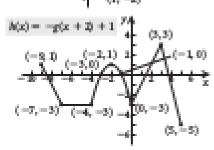
107.



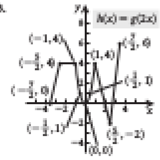
109.



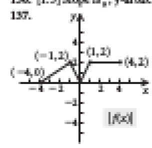
111.



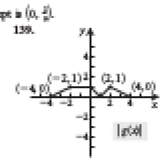
113.



115. (f) 117. (f) 119. (d) 121. (c)
123. $f(-x) = 2(-x)^4 - 35(-x)^3 + 3(-x) - 5 = 2x^4 + 35x^3 - 3x - 5 = g(x)$
125. $g(x) = x^3 - 3x^2 + 2$
127. $k(x) = (x+1)^3 - 3(x+1)^2$
129. Discussion and Writing 131. Discussion and Writing
133. [1.2] (a) 38; (b) 38; (c) $5a^2 - 7$; (d) $5a^2 - 7$
134. [1.2] (a) 22; (b) −22; (c) $4a^3 - 5a$; (d) $-4a^3 + 5a$
135. [1.3] $y = -\frac{1}{8}x + \frac{7}{8}$
136. [1.3] Slope is $\frac{2}{9}$; y-intercept is $\left(0, \frac{1}{9}\right)$.
137.



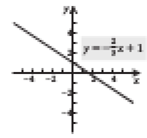
139.



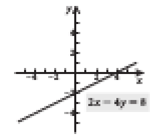
141. Start with the graph of $g(x) = [\![x]\!]$. Shift it right $\frac{1}{2}$ unit. Domain: all real numbers; range: all integers.
143. Odd 145. x-axis, yes; y-axis, no; origin, no
147. x-axis, yes; y-axis, no; origin, no 149. 5
151. $(3, 0)$; $(5, 6)$; $\left(\frac{7}{2}, 4\right)$ 153. True
155. $E(-x) = \dfrac{f(-x) + f(-(-x))}{2} = \dfrac{f(-x) + f(x)}{2} = E(x)$
157. (a) $E(x) + O(x) = \dfrac{f(x) + f(-x)}{2} + \dfrac{f(x) - f(-x)}{2} = \dfrac{2f(x)}{2} = f(x)$; (b) $f(x) = \dfrac{-22x^2 + \sqrt{x} + \sqrt{-x} - 20}{2} + \dfrac{8x^3 + \sqrt{x} - \sqrt{-x}}{2}$

Review Exercises: Chapter 1

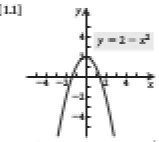
1. [1.1] Yes; no 2. [1.1] Yes; no
3. [1.1]



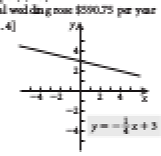
4. [1.1]



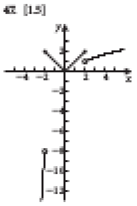
5. [1.1]



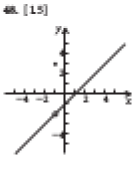
6. [1.1] $\sqrt{34} = 5.831$ 7. [1.1] $\left(\frac{1}{2}, \frac{11}{2}\right)$
8. [1.1] $(x+2)^2 + (y-6)^2 = 13$
9. [1.1] Center: $(-1, 3)$; radius: 4
10. [1.1] Center: $(3, -5)$; radius: 1
11. [1.1] $(x-2)^2 + (y-4)^2 = 26$
12. [1.2] Not a function; domain: {3, 5, 7}; range: {1, 3, 5, 7}
13. [1.2] Function; domain: {−2, 0, 1, 2, 7}; range: {−7, −4, −2, 2, 7} 14. [1.2] No 15. [1.2] Yes
16. [1.2] No 17. [1.2] Yes 18. [1.2] $f(2) = -1$; $f(-3) = -1$; $f(0) = -1$ 19. [1.2] All real numbers
20. [1.2] $\{x \mid x \neq 0\}$ 21. [1.2] $\{x \mid x \neq 5 \text{ and } x \neq 1\}$
22. [1.2] $\{x \mid x \neq -4 \text{ and } x \neq 4\}$
23. [1.2] Domain: $[-4, 4]$; range: $[0, 4]$
24. [1.2] Domain: $(-\infty, \infty)$; range: $[0, \infty)$
25. [1.2] Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$
26. [1.2] Domain: $(-\infty, \infty)$; range: $[0, \infty)$
27. [1.2] (a) −3; (b) 9; (c) $a^2 - 3a - 1$; [1.6] (d) $2x + h - 1$ 28. [1.3] (a) Yes; (b) no; (c) no, strictly speaking, but data might be modeled by a linear regression function. 29. [1.3] (a) Yes; (b) yes; (c) yes
30. [1.3] $\frac{4}{5}$ 31. [1.3] 0 32. [1.3] Not defined
33. [1.4] $m = -2$; y-intercept: $(0, -7)$
34. [1.3] The cost of a formal wedding rose \$590.75 per year from 1995 to 2003. 35. [1.4]



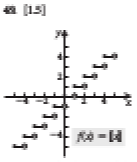
36. [1.4] $y = -\frac{2}{3}x - 4$ 37. [1.4] $y = 3x + 5$
38. [1.4] $y = \frac{1}{3}x - \frac{1}{3}$ 39. [1.4] Parallel
40. [1.4] Neither 41. [1.4] Perpendicular
42. [1.4] $y = -\frac{2}{3}x - \frac{1}{3}$ 43. [1.4] $y = \frac{3}{2}x - \frac{5}{2}$
44. [1.3] $C(t) = 25 + 20t$; \$145
45. [1.3] (a) 70°C, 220°C, 10,020°C; (b) [0, 5600]
46. [1.4] (a) Using (10, 16.5) and (50, 58.3), $y = 1.045x + 6.05$; 68.75 million. Answers may vary; (b) $y = 0.9406550062x + 9.457109283$; 66.4 million; $r = 0.9967$, a good fit
47. [1.5]



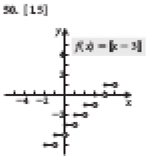
48. [1.5]



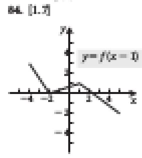
49. [1.5]



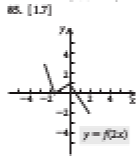
50. [1.5]



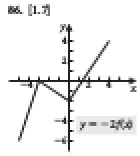
51. [1.5] $f(-1) = 1$; $f(5) = 2$; $f(-2) = 2$; $f(-3) = -27$
52. [1.5] $A(x) = 2x\sqrt{4 - x^2}$
53. [1.5] (a) $A(x) = x^2 + \dfrac{432}{x}$; (b) $(0, \infty)$; (c) $x = 6$ in.; height $= 3$ in.
54. [1.6] −33 55. [1.6] 0 56. [1.6] Does not exist
57. [1.6] (a) Domain of f: $\{x \mid x \neq 0\}$; domain of g: all real numbers; domain of $f + g$, $f - g$, and fg: $\{x \mid x \neq 0\}$; domain of f/g: $\left\{x \,\middle|\, x \neq 0 \text{ and } x \neq \dfrac{3}{2}\right\}$;
(b) $(f+g)(x) = \dfrac{4}{x^2} + 3 - 2x$; $(f-g)(x) = \dfrac{4}{x^2} - 3 + 2x$; $(fg)(x) = \dfrac{12}{x^2} - \dfrac{8}{x}$; $(f/g)(x) = \dfrac{4}{x^2(3 - 2x)}$
58. [1.6] (a) Domain of f, g, $f + g$, $f - g$, and fg: all real numbers; domain of f/g: $\left\{x \,\middle|\, x \neq \dfrac{1}{2}\right\}$;
(b) $(f+g)(x) = 3x^2 + 6x - 1$; $(f-g)(x) = 3x^2 + 2x + 1$; $(fg)(x) = 6x^3 + 5x^2 - 4x$; $(f/g)(x) = \dfrac{3x^2 + 4x}{2x - 1}$
59. [1.6] $P(x) = -0.5x^2 + 105x - 6$ 60. [1.6] $-2x - h$
61. [1.6] (a) Domain of $f \circ g$: $\left\{x \,\middle|\, x \neq \frac{3}{2}\right\}$; domain of $g \circ f$: $\{x \mid x \neq 0\}$; (b) $(f \circ g)(x) = \dfrac{4}{(3 - 2x)^2}$; $(g \circ f)(x) = 3 - \dfrac{8}{x^2}$
62. [1.6] (a) Domain of $f \circ g$ and $g \circ f$: all real numbers; (b) $(f \circ g)(x) = 12x^2 - 4x - 1$; $(g \circ f)(x) = 6x^2 + 8x - 1$
63. [1.6] $f(x) = \sqrt{x}$, $g(x) = 5x + 2$. Answers may vary.
64. [1.6] $f(x) = 4x^2 + 9$, $g(x) = 5x - 1$. Answers may vary.
65. [1.7] x-axis, yes; y-axis, yes; origin, yes
66. [1.7] x-axis, yes; y-axis, yes; origin, yes
67. [1.7] x-axis, no; y-axis, no; origin, no
68. [1.7] x-axis, no; y-axis, yes; origin, no
69. [1.7] x-axis, no; y-axis, no; origin, yes
70. [1.7] x-axis, no; y-axis, yes; origin, no
71. [1.7] Even 72. [1.7] Even 73. [1.7] Odd
74. [1.7] Even 75. [1.7] Even 76. [1.7] Neither
77. [1.7] Odd 78. [1.7] Even 79. [1.7] Even
80. [1.7] Odd 81. [1.7] $f(x) = (x+3)^2$
82. [1.7] $f(x) = -\sqrt{x - 3} + 4$ 83. [1.7] $f(x) = 2|x - 3|$
84. [1.7]



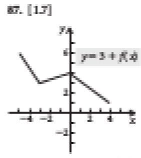
85. [1.7]



86. [1.7]



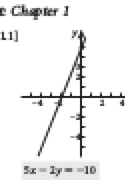
87. [1.7]



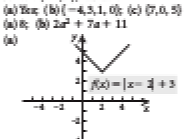
88. Discussion and Writing [1.7] In the graph of $y = f(cx)$, the constant c stretches or shrinks the graph of $y = f(x)$ horizontally. The constant c in $y = cf(x)$ stretches or shrinks the graph of $y = f(x)$ vertically. For $y = f(cx)$, the x-coordinates of $y = f(x)$ are divided by c; for $y = cf(x)$, the y-coordinates of $y = f(x)$ are multiplied by c.
89. Discussion and Writing [1.7] (a) To draw the graph of y_2 from the graph of y_1, reflect across the x-axis the portions of the graph for which the y-coordinates are negative. (b) To draw the graph of y_3 from the graph of y_1, draw the portion of the graph of y_1 to the right of the y-axis; then draw its reflection across the y-axis. 90. [1.2] $\{x \mid x < 0\}$
91. [1.2] $\{x \mid x \neq -3 \text{ and } x \neq 0 \text{ and } x \neq 3\}$
92. [1.6], [1.7] Let $f(x)$ and $g(x)$ be odd functions. Then by definition, $f(-x) = -f(x)$, or $f(x) = -f(-x)$, and $g(-x) = -g(x)$, or $g(x) = -g(-x)$. Thus, $(f+g)(x) = f(x) + g(x) = -f(-x) + [-g(-x)] = -[f(-x) + g(-x)] = -(f+g)(-x)$ and $f + g$ is odd.
93. [1.7] Reflect the graph of $y = f(x)$ across the x-axis and then across the y-axis.

Test: Chapter 1

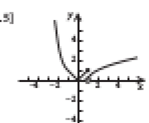
1. [1.1]



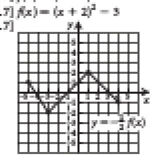
2. [1.1] $\sqrt{45} \approx 6.708$
3. [1.1] $\left(-3, \frac{9}{2}\right)$ 4. [1.1] $(x+1)^2 + (y-2)^2 = 5$
5. [1.1] Center: $(-4, 5)$; radius: 6
6. [1.2] (a) Yes; (b) {−4, 3, 1, 0}; (c) {7, 0, 5}
7. [1.2] (a) 8; (b) $2a^2 + 7a + 11$
8. [1.2] (a)



(b) $(-\infty, \infty)$; (c) $[3, \infty)$
9. [1.2] $\{x \mid x \neq 4\}$, or $(-\infty, 4) \cup (4, \infty)$
10. [1.2] $(-\infty, \infty)$ 11. [1.2] $\{x \mid -5 \le x \le 5\}$, or $[-5, 5]$
12. [1.2] (a) No; (b) yes 13. [1.3] Not defined
14. [1.3] $-\frac{11}{6}$ 15. [1.3] 0 16. [1.3] Debit-card transactions increased approximately \$16.6 billion per year from 1990 to 1999. 17. [1.4] Slope: $\frac{3}{2}$; y-intercept: $\left(0, \frac{5}{2}\right)$
18. [1.4] $y = -\frac{5}{8}x - 5$ 19. [1.4] $y = -\frac{3}{4}x + \frac{1}{4}$
20. [1.4] $y = -\frac{1}{2}x + \frac{3}{2}$ 21. [1.4] Perpendicular
22. [1.4] Using (0, 23.3) and (2, 25.1), $y = 0.9x + 23.3$; 31.4 gal; (b) $y = 1.02x + 23.12$, 32.3 gal; $r = 0.9898$
23. [1.5]



24. [1.5] $f\left(-\frac{7}{8}\right) = \frac{7}{8}$; $f(5) = 2$; $f(-4) = 16$
25. [1.6] $(f - g)(-1) = 6$
26. [1.6] (a) $(-\infty, \infty)$; (b) $[3, \infty)$; (c) $(f - g)(x) = x^2 - \sqrt{x - 3}$; (d) $(fg)(x) = x^2\sqrt{x - 3}$; (e) $(3, \infty)$
27. [1.6] $f(x) = x^4$; $g(x) = 2x - 7$ 28. [1.6] $2x + h$
29. [1.6] $(f \circ g)(x) = \sqrt{x^2 - 4}$; $(g \circ f)(x) = x - 4$
30. [1.6] Domain of $(f \circ g)(x) = (-\infty, -2) \cup (2, \infty)$; domain of $(g \circ f)(x) = [5, \infty)$ 31. [1.7] x-axis: no; y-axis: yes; origin: no 32. [1.7] Odd
33. [1.7] $f(x) = (x - 2)^2 - 1$
34. [1.7] $f(x) = (x + 2)^2 - 3$
35. [1.7]



36. [1.7] $(-1, 1)$

Chapter 2

Exercise Set 2.1

1. 4 3. $-\frac{3}{4}$ 5. −9 7. $\frac{11}{5}$ 9. 8 11. −4 13. 6
15. −1 17. $\frac{4}{5}$ 19. $-\frac{3}{2}$ 21. $-\frac{4}{5}$ 23. $\frac{2}{5}$
25. 34,527 hybrid vehicles 27. About 9.2%
29. 29.2 million households 31. 2080 calories
33. \$1300 35. \$9800 37. 12 mi 39. 26°, 130°, 24°
41. Length: 93 m; width: 68 m 43. Length: 100 yd; width: 65 yd 45. 67.5 lb 47. Freight: 66 mph; passenger: 80 mph 49. 4.5 hr 51. 1.75 hr 53. \$2400 at 3%; \$2600 at 4% 55. I: about 1724 violations; II: about 265 violations 57. AOL: 52.5 million users; Microsoft: 29.4 million users 59. 640 yr 61. −5
63. 18 65. 16 67. −12 69. 6 71. 20 73. 6
75. 15 77. (a) (4, 0); (b) 4 79. (a) (−2, 0); (b) −2
81. (a) (−4, 0); (b) −4 83. $b = \dfrac{2A}{h}$ 85. $w = \dfrac{P - 2l}{2}$
87. $h = \dfrac{2A}{b_1 + b_2}$ 89. $\pi = \dfrac{3V}{4r^3}$ 91. $C = \dfrac{5}{9}(F - 32)$