Algebra in-class worksheet, Chapter 1.5
More on Functions

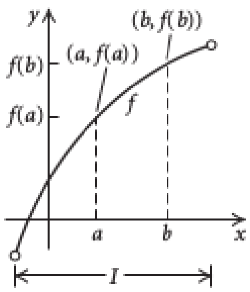
Section 1-5 in a nutshell:

Increasing, Decreasing, and Constant Functions

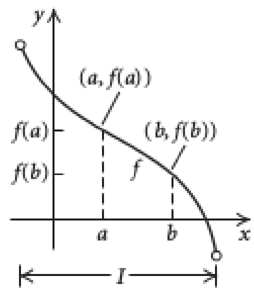
A function f is said to be **increasing** on an *open* interval I, if for all a and b in that interval, $a < b$ implies $f(a) < f(b)$. (See Fig. 1.)

A function f is said to be **decreasing** on an *open* interval I, if for all a and b in that interval, $a < b$ implies $f(a) > f(b)$. (See Fig. 2.)

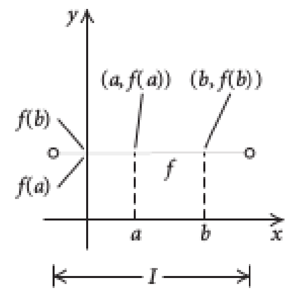
A function f is said to be **constant** on an *open* interval I, if for all a and b in that interval, $f(a) = f(b)$. (See Fig. 3.)



For $a < b$ in I, $f(a) < f(b)$; f is *increasing* on I.



For $a < b$ in I, $f(a) > f(b)$; f is *decreasing* on I.



For all a and b in I, $f(a) = f(b)$; f is *constant* on I.

Relative Maxima and Minima

Suppose that f is a function for which $f(c)$ exists for some c in the domain of f. Then:

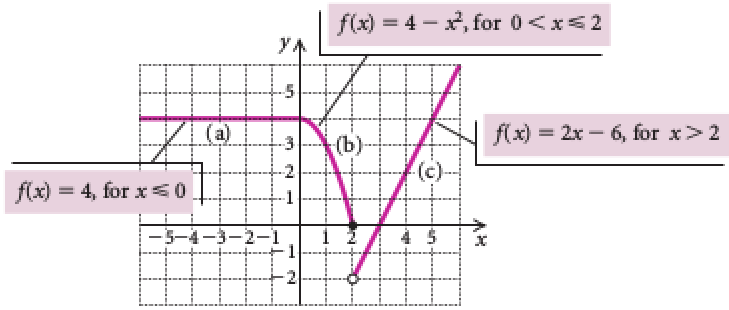
$f(c)$ is a **relative maximum** if there exists an *open* interval I containing c such that $f(c) > f(x)$, for all x in I where $x \neq c$; and

$f(c)$ is a **relative minimum** if there exists an *open* interval I containing c such that $f(c) < f(x)$, for all x in I where $x \neq c$.

Functions Defined Piecewise

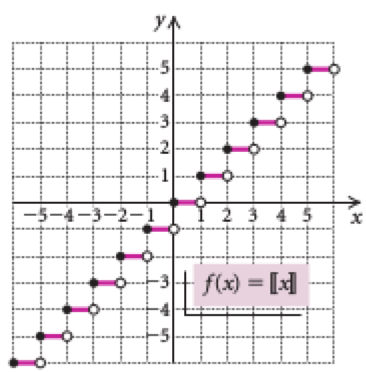
Sometimes functions are defined **piecewise** using different output formulas for different parts of the domain.

$$f(x) = \begin{cases} 4, & \text{for } x \le 0, \\ 4 - x^2, & \text{for } 0 < x \le 2, \\ 2x - 6, & \text{for } x > 2. \end{cases}$$



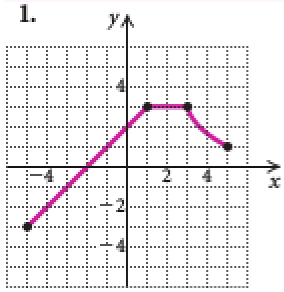
> ***Greatest Integer Function***
> $f(x) = [\![x]\!] =$ the greatest integer *less than or equal to x.*

$$f(x) = [\![x]\!] = \begin{cases} \vdots \\ -3, & \text{for } -3 \le x < -2, \\ -2, & \text{for } -2 \le x < -1, \\ -1, & \text{for } -1 \le x < 0, \\ 0, & \text{for } 0 \le x < 1, \\ 1, & \text{for } 1 \le x < 2, \\ 2, & \text{for } 2 \le x < 3, \\ 3, & \text{for } 3 \le x < 4, \\ \vdots \end{cases}$$



Determine for each graphed function:
1) the intervals on which the function is increasing, decreasing, and constant
2) the domain and range

1.



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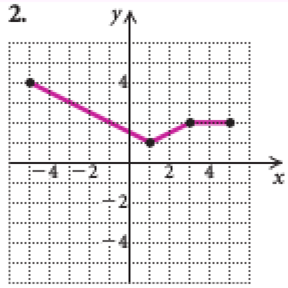

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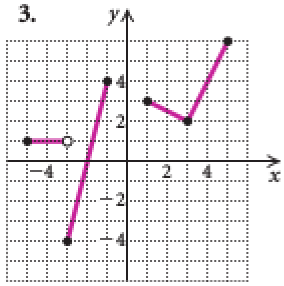

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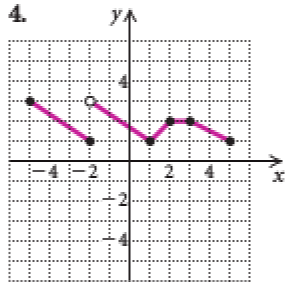

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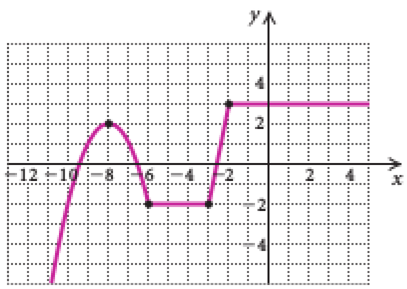

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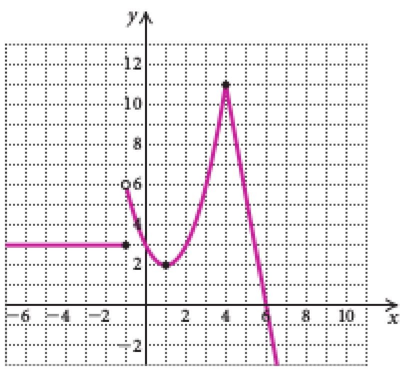
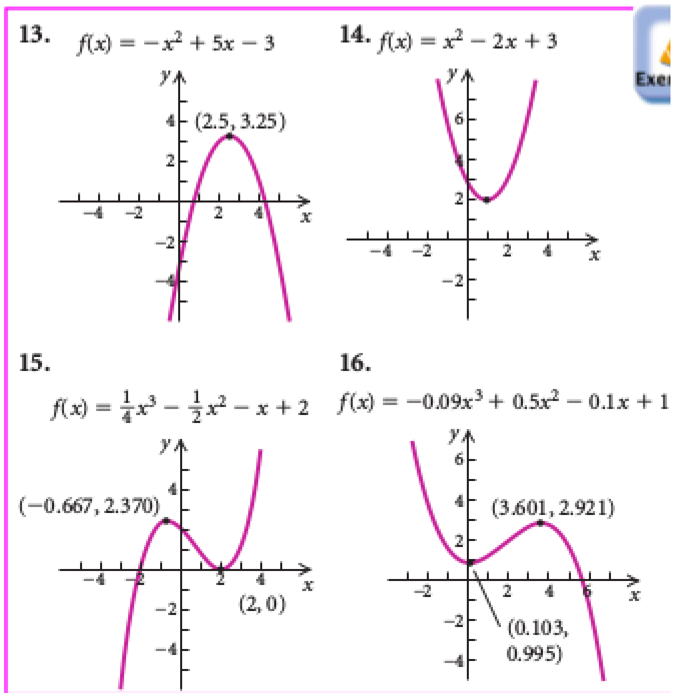


Determine any relative maxima or minima of the graphed functons.
Determine the intervals where each function is increasing or decreasing



Determine the intervals on which each function is increasing or decreasing.
Determine any relative maxima or minima

17. $f(x) = x^2$

18. $f(x) = 4 - x^2$

19. $f(x) = 5 - |x|$

20. $f(x) = |x + 3| - 5$

21. $f(x) = x^2 - 6x + 10$

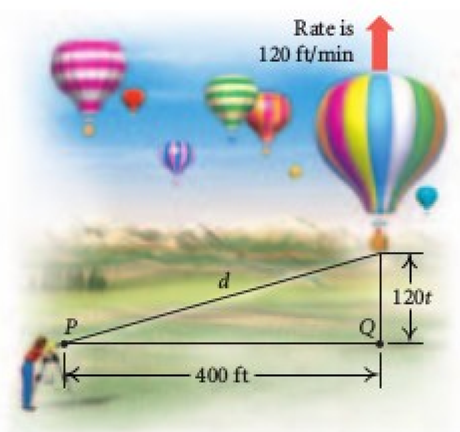
22. $f(x) = -x^2 - 8x - 9$

Story Problems:

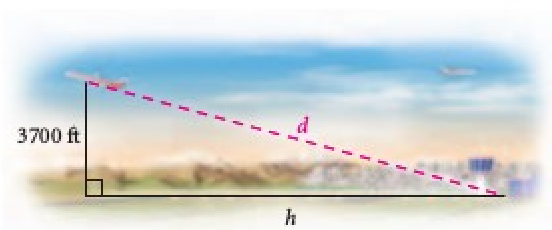
33. *Garden Area.* Creative Landscaping has 60 yd of fencing with which to enclose a rectangular flower garden. If the garden is x yards long, express the garden's area as a function of the length.

34. *Triangular Flag.* A scout troop is designing a triangular flag so that the length of its base is 7 in. less than twice the height, h. Express the area of the flag as a function of the height.

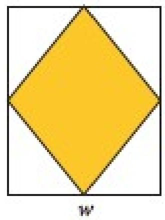
35. *Rising Balloon.* A hot-air balloon rises straight up from the ground at a rate of 120 ft/min. The balloon is tracked from a rangefinder on the ground at point P, which is 400 ft from the release point Q of the balloon. Let $d =$ the distance from the balloon to the rangefinder and $t =$ the time, in minutes, since the balloon was released. Express d as a function of t.



36. *Airplane Distance.* An airplane is flying at an altitude of 3700 ft. The slanted distance directly to the airport is d feet. Express the horizontal distance h as a function of d.

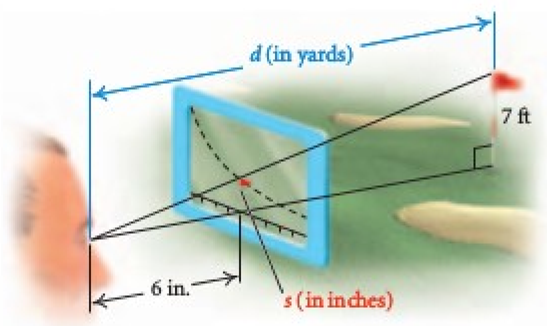


37. *Inscribed Rhombus.* A rhombus is inscribed in a rectangle that is w meters wide with a perimeter of 40 m. Each vertex of the rhombus is a midpoint of a side of the rectangle. Express the area of the rhombus as a function of the rectangle's width.

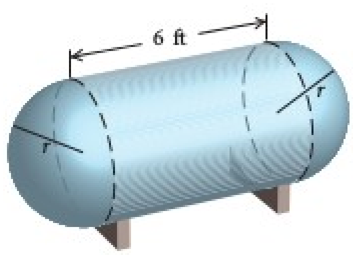


38. *Tablecloth Area.* A tailor uses 16 ft of lace to trim the edges of a rectangular tablecloth. If the tablecloth is w feet wide, express its area as a function of the width.

39. *Golf Distance Finder.* A device used in golf to estimate the distance d, in yards, to a hole measures the size s, in inches, that the 7-ft pin appears to be in a viewfinder. Express the distance d as a function of s.

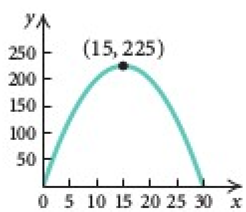


40. *Gas Tank Volume.* A gas tank has ends that are hemispheres of radius r ft. The cylindrical midsection is 6 ft long. Express the volume of the tank as a function of r.



41. *Play Space.* A daycare center has 30 ft of dividers with which to enclose a rectangular play space in a corner of a large room. The sides against the wall require no partition. Suppose the play space is x feet long.

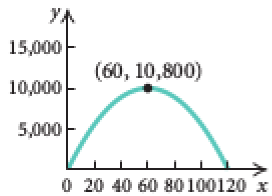
a) Express the area of the play space as a function of x.
b) Find the domain of the function.
c) Using the graph shown below, determine the dimensions that yield the maximum area.



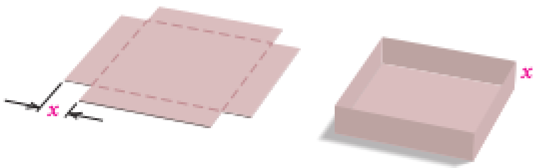
42. *Corral Design.* A rancher has 360 yd of fencing with which to enclose two adjacent rectangular corrals, one for sheep and one for cattle. A river forms one side of the corrals. Suppose the width of each corral is x yards.

a) Express the total area of the two corrals as a function of x.
b) Find the domain of the function.

c) Using the graph shown below, determine the dimensions that yield the maximum area.

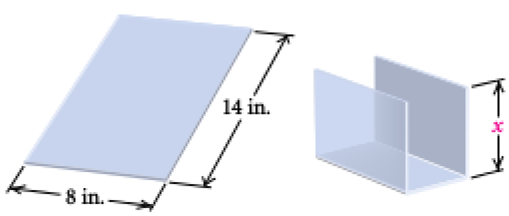


43. *Volume of a Box.* From a 12-cm by 12-cm piece of cardboard, square corners are cut out so that the sides can be folded up to make a box.



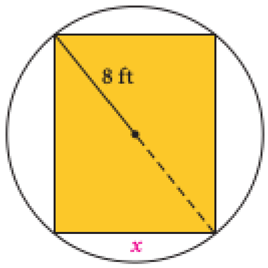
a) Express the volume of the box as a function of the length x, in centimeters, of a cut-out square.
b) Find the domain of the function.
c) Graph the function with a graphing calculator.
d) What dimensions yield the maximum volume?

44. *Molding Plastics.* Plastics Unlimited plans to produce a one-component vertical file by bending the long side of an 8-in. by 14-in. sheet of plastic along two lines to form a U shape.



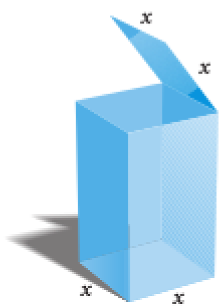
a) Express the volume of the file as a function of the height x, in inches, of the file.
b) Find the domain of the function.
c) Graph the function with a graphing calculator.
d) How tall should the file be in order to maximize the volume that the file can hold?

45. *Area of an Inscribed Rectangle.* A rectangle that is x feet wide is inscribed in a circle of radius 8 ft.



a) Express the area of the rectangle as a function of x.
b) Find the domain of the function.
c) Graph the function with a graphing calculator.
d) What dimensions maximize the area of the rectangle?

46. *Cost of Material.* A rectangular box with volume $320\ \text{ft}^3$ is built with a square base and top. The cost is $\$1.50/\text{ft}^2$ for the bottom, $\$2.50/\text{ft}^2$ for the sides, and $\$1/\text{ft}^2$ for the top. Let $x =$ the length of the base, in feet.



a) Express the cost of the box as a function of x.
b) Find the domain of the function.
c) Graph the function with a graphing calculator.
d) What dimensions minimize the cost of the box?

Evaluate the following piecewise functions at the specified values

47. $g(x) = \begin{cases} x + 4, & \text{for } x \leq 1, \\ 8 - x, & \text{for } x > 1 \end{cases}$

$g(-4)$, $g(0)$, $g(1)$, and $g(3)$

48. $f(x) = \begin{cases} 3, & \text{for } x \leq -2, \\ \frac{1}{2}x + 6, & \text{for } x > -2 \end{cases}$

$f(-5)$, $f(-2)$, $f(0)$, and $f(2)$

49. $h(x) = \begin{cases} -3x - 18, & \text{for } x < -5, \\ 1, & \text{for } -5 \leq x < 1, \\ x + 2, & \text{for } x \geq 1 \end{cases}$

$h(-5)$, $h(0)$, $h(1)$, and $h(4)$

50. $f(x) = \begin{cases} -5x - 8, & \text{for } x < -2, \\ \frac{1}{2}x + 5, & \text{for } -2 \leq x \leq 4, \\ 10 - 2x, & \text{for } x > 4 \end{cases}$

$f(-4)$, $f(-2)$, $f(4)$, and $f(6)$

Make a hand-drawn graph of each of the following functions.
Find their domain and range.

51. $f(x) = \begin{cases} \frac{1}{2}x, & \text{for } x < 0, \\ x + 3, & \text{for } x \geq 0 \end{cases}$

52. $f(x) = \begin{cases} -\frac{1}{3}x + 2, & \text{for } x \leq 0, \\ x - 5, & \text{for } x > 0 \end{cases}$

53. $f(x) = \begin{cases} -\frac{3}{4}x + 2, & \text{for } x < 4, \\ -1, & \text{for } x \geq 4 \end{cases}$

54. $f(x) = \begin{cases} 4, & \text{for } x \leq -2, \\ x + 1, & \text{for } -2 < x < 3, \\ -x, & \text{for } x \geq 3 \end{cases}$

55. $f(x) = \begin{cases} x + 1, & \text{for } x \leq -3, \\ -1, & \text{for } -3 < x < 4, \\ \frac{1}{2}x, & \text{for } x \geq 4 \end{cases}$

56. $f(x) = \begin{cases} \dfrac{x^2 - 9}{x + 3}, & \text{for } x \neq -3, \\ 5, & \text{for } x = -3 \end{cases}$

57. $f(x) = \begin{cases} 2, & \text{for } x = 5, \\ \dfrac{x^2 - 25}{x - 5}, & \text{for } x \neq 5 \end{cases}$

58. $f(x) = \begin{cases} \dfrac{x^2 + 3x + 2}{x + 1}, & \text{for } x \neq -1, \\ 7, & \text{for } x = -1 \end{cases}$

59. $f(x) = [\![x]\!]$

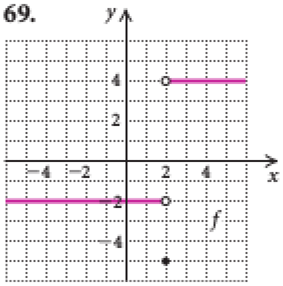
60. $f(x) = 2[\![x]\!]$

61. $g(x) = 1 + [\![x]\!]$

62. $h(x) = \frac{1}{2}[\![x]\!] - 2$

Determine the domain and range for each function. Then write as an equation

69.



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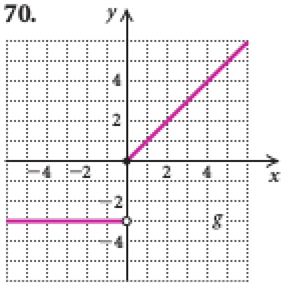

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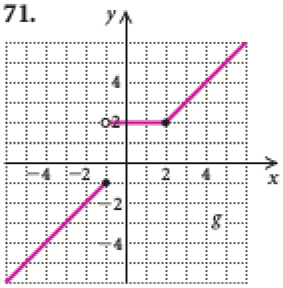

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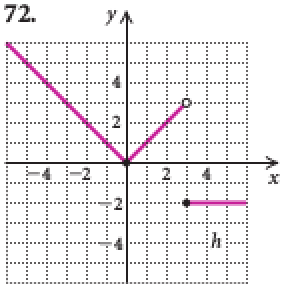

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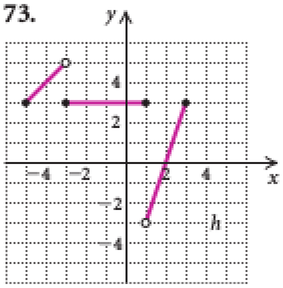

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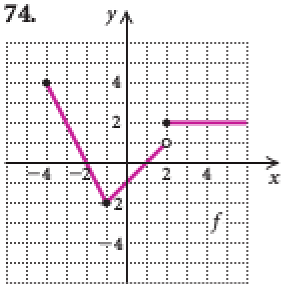
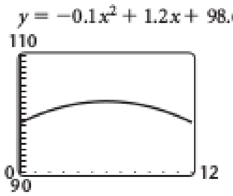
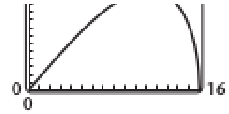


Exercise Set 1.5

1. **(a)** $(-5, 1)$; **(b)** $(3, 5)$; **(c)** $(1, 3)$
3. **(a)** $(-3, -1), (3, 5)$; **(b)** $(1, 3)$; **(c)** $(-5, -3)$
5. **(a)** $(-\infty, -8), (-3, -2)$; **(b)** $(-8, -6)$;
(c) $(-6, -3), (-2, \infty)$
7. Domain: $[-5, 5]$; range: $[-3, 3]$
9. Domain: $[-5, -1] \cup [1, 5]$; range: $[-4, 6]$
11. Domain: $(-\infty, \infty)$; range: $(-\infty, 3]$
13. Relative maximum: 3.25 at $x = 2.5$; increasing: $(-\infty, 2.5)$; decreasing: $(2.5, \infty)$
15. Relative maximum: 2.370 at $x = -0.667$; relative minimum: 0 at $x = 2$; increasing: $(-\infty, -0.667), (2, \infty)$; decreasing: $(-0.667, 2)$
17. Increasing: $(0, \infty)$; decreasing: $(-\infty, 0)$; relative minimum: 0 at $x = 0$
19. Increasing: $(-\infty, 0)$; decreasing: $(0, \infty)$; relative maximum: 5 at $x = 0$
21. Increasing: $(3, \infty)$; decreasing: $(-\infty, 3)$; relative minimum: 1 at $x = 3$
23. Increasing: $(1, 3)$; decreasing: $(-\infty, 1), (3, \infty)$; relative maximum: -4 at $x = 3$; relative minimum: -8 at $x = 1$
25. Increasing: $(-1.552, 0), (1.552, \infty)$; decreasing: $(-\infty, -1.552), (0, 1.552)$; relative maximum: 4.07 at $x = 0$; relative minima: -2.314 at $x = -1.552$, -2.314 at $x = 1.552$
27. **(a)** $y = -0.1x^2 + 1.2x + 98.6$

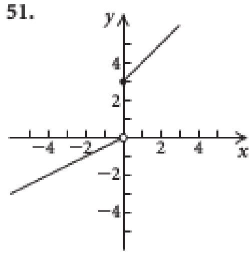


(b) 6 days after the illness began, the temperature is 102.2°F.



(d) 11.314 ft by 11.314 ft
47. $g(-4) = 0$; $g(0) = 4$; $g(1) = 5$; $g(3) = 5$
49. $h(-5) = 1$; $h(0) = 1$; $h(1) = 3$; $h(4) = 6$

51.



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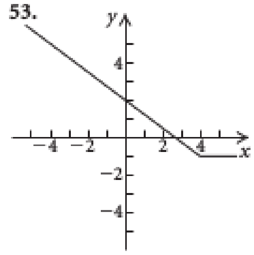

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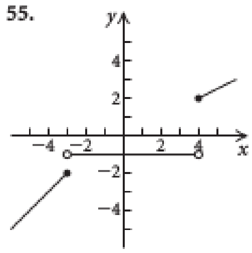

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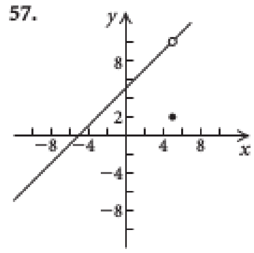

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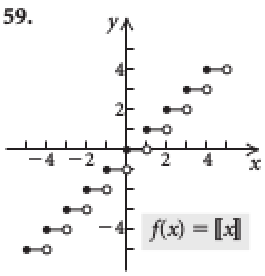

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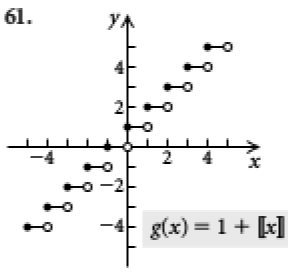


63. Domain: $(-\infty, \infty)$; range: $(-\infty, 0) \cup [3, \infty)$
65. Domain: $(-\infty, \infty)$; range: $[-1, \infty)$
67. Domain: $(-\infty, \infty)$;
range: $\{y \mid y \le -2 \text{ or } y = -1 \text{ or } y \ge 2\}$
69. Domain: $(-\infty, \infty)$; range: $\{-5, -2, 4\}$

$$f(x) = \begin{cases} -2, & \text{for } x < 2, \\ -5, & \text{for } x = 2, \\ 4, & \text{for } x > 2 \end{cases}$$

71. Domain: $(-\infty, \infty)$; range: $(-\infty, -1] \cup [2, \infty)$;

$$g(x) = \begin{cases} x, & \text{for } x \le -1, \\ 2, & \text{for } -1 < x < 2, \\ x, & \text{for } x \ge 2 \end{cases}$$

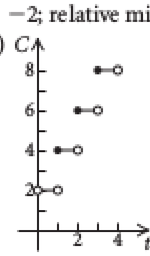
or

$$g(x) = \begin{cases} x, & \text{for } x \le -1, \\ 2, & \text{for } -1 < x \le 2, \\ x, & \text{for } x > 2 \end{cases}$$

73. Domain: $[-5, 3]$; range: $(-3, 5)$;

$$h(x) = \begin{cases} x + 8, & \text{for } -5 \le x < -3, \\ 3, & \text{for } -3 \le x \le 1, \\ 3x - 6, & \text{for } 1 < x \le 3 \end{cases}$$

75. Discussion and Writing **77.** [1.2] Function; domain; range; domain; exactly one; range
78. [1.1] Midpoint formula **79.** [1.1] x-intercept
80. [1.3] Constant; identity **81.** Increasing: $(-5, -2)$, $(4, \infty)$; decreasing: $(-\infty, -5)$, $(-2, 4)$; relative maximum: 560 at $x = -2$; relative minima: 425 at $x = -5$, -304 at $x = 4$
83. **(a)**



(b) $C(t) = 2([\![t]\!] + 1), t > 0$
85. $\{x \mid -5 \le x < -4 \text{ or } 5 \le x < 6\}$
87. **(a)** $h(r) = \dfrac{30 - 5r}{3}$; **(b)** $V(r) = \pi r^2 \left(\dfrac{30 - 5r}{3}\right)$;
(c) $V(h) = \pi h \left(\dfrac{30 - 3h}{5}\right)^2$