Algebra in-class worksheet, Chapter 1.3
Slope and Linear Functions

Linear Functions

A function f is a **linear function** if it can be written as

$$f(x) = mx + b,$$

where m and b are constants.

(If $m = 0$, the function is a **constant function** $f(x) = b$. If $m = 1$ and $b = 0$, the function is the **identity function** $f(x) = x$.)

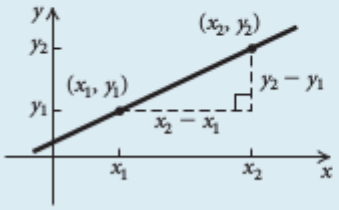
Horizontal and Vertical Lines

Horizontal lines are given by equations of the type $y = b$ or $f(x) = b$. (They are functions.)

Vertical lines are given by equations of the type $x = a$. (They are *not* functions.)

Slope

The **slope** m of a line containing points (x_1, y_1) and (x_2, y_2) is given by



$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{the change in } y}{\text{the change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}.$$

Horizontal and Vertical Lines

If a line is horizontal, the change in y for any two points is 0 and the change in x is nonzero. Thus a horizontal line has slope 0 (see Fig. 4 on the next page).

If a line is vertical, the change in y for any two points is nonzero and the change in x is 0. Thus the slope is *not defined* because we cannot divide by 0 (see Fig. 5).

In Exercises 1–4, the table of data contains input–output values for a function. Answer the following questions for each table.

a) *Is the change in the inputs x the same?*
b) *Is the change in the outputs y the same?*
c) *Is the function linear?*

1.

x	y
−3	7
−2	10
−1	13
0	16
1	19
2	22
3	25

2.

x	y
20	12.4
30	24.8
40	49.6
50	99.2
60	198.4
70	396.8
80	793.6

3.

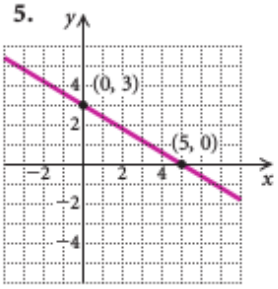
x	y
11	3.2
26	5.7
41	8.2
56	9.3
71	11.3
86	13.7
101	19.1

4.

x	y
2	−8
4	−12
6	−16
8	−20
10	−24
12	−28
14	−32

Find the slope of the line containing the given points.

5.



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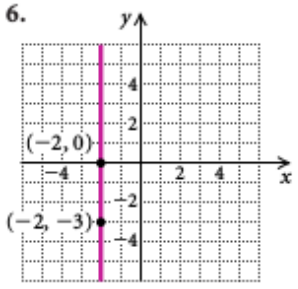

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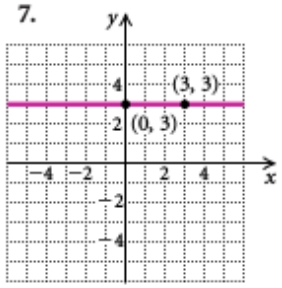
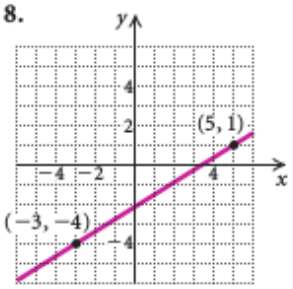


8.



9. $(9,4)$ and $(-1,2)$

10. $(-3,7)$ and $(5,-1)$

11. $(4,-9)$ and $(-5,6)$

12. $(-6,-1)$ and $(2,-13)$

13. $(0.7,-0.1)$ and $(-0.3,-0.4)$

14. $\left(-\frac{3}{4},-\frac{1}{4}\right)$ and $\left(\frac{2}{7},-\frac{5}{7}\right)$

15. $(2,-2)$ and $(4,-2)$

16. $(-9,8)$ and $(7,-6)$

17. $\left(\frac{1}{2},-\frac{3}{5}\right)$ and $\left(-\frac{1}{2},\frac{3}{5}\right)$

18. $(-8.26,4.04)$ and $(3.14,-2.16)$

19. $(16,-13)$ and $(-8,-5)$

20. $(-10,-7)$ and $(-10,7)$

21. $(\pi,-3)$ and $(\pi,2)$

22. $\left(\sqrt{2},-4\right)$ and $(0.56,-4)$

Graph the linear equation and determine its slope, if it exists.

23. $y=-\frac{1}{2}x+3$

24. $y=\frac{3}{2}x-4$

25. $2y-3x=-6$

26. $x+2y=1$

27. $5x+2y=10$

28. $2y-x=8$

29. $y=-\frac{2}{3}$

30. $x=3$

Determine the slope, if it exists, of the graph of the given linear equation.

31. $y = 1.3x - 5$

32. $y = -\frac{2}{5}x + 7$

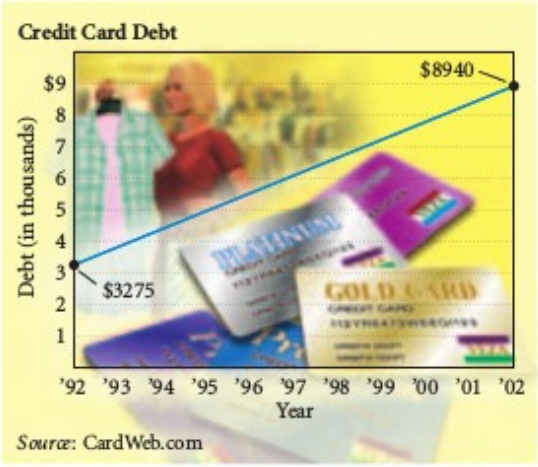
33. $x = -2$

34. $3x - 4y = -11$

35. $10y + x = 9$

36. $y = \frac{3}{4}$

37. *Credit Card Debt.* In one decade, the average household's credit card balance rose 173%. Find the average rate of change in the credit card balance from 1992 to 2002.



38. *Unwanted Commercial E-mail.* Between 1999 and 2002, the number of unwanted commercial e-mail messages, called spam, sent daily rose 460%. Find the average rate of change in delivery of spam from 1999 to 2002.

39. *Overseas Adoptions.* Overseas adoptions by U.S. parents have increased by more than 200% from 1992 to 2002. In 1992, 6472 visas were issued to orphans from other countries. In 2002, 20,099 visas were issued. Find the average rate of change in the number of overseas adoptions over the 10-year period. (*Source*: National Adoption Information Clearinghouse)

40. *Chinese Adoptions.* In 1992, 210 children from mainland China were adopted by U.S. parents. The number of Chinese children adopted ten years later, in 2002, was 6062. Find the average rate of change in the number of Chinese adoptions from 1992 to 2002. (*Source*: Bureau of Citizenship and Immigration Services)

41. *Running Rate.* An Olympic marathoner passes the 10-km point of a race after 50 min and arrives at the 25-km point $1\frac{1}{2}$ hr later. Find the speed (average rate of change) of the runner.

42. *Work Rate.* As a typist resumes work on a research paper, $\frac{1}{6}$ of the paper has already been keyboarded. Six hours later, the paper is $\frac{3}{4}$ done. Calculate the worker's typing rate.

43. *Ideal Minimum Weight.* One way to estimate the ideal minimum weight of a woman in pounds is to multiply her height in inches by 4 and subtract 130. Let W = the ideal minimum weight and h = height.

a) Express W as a linear function of h.

b) Graph W.

c) Find the ideal minimum weight of a woman whose height is 62 in.

d) Find the domain of the function.

44. *Pressure at Sea Depth.* The function P, given by

$$P(d) = \frac{1}{33}d + 1,$$

gives the pressure, in atmospheres (atm), at a depth d, in feet, under the sea.

a) Graph P.
b) Find $P(0)$, $P(5)$, $P(10)$, $P(33)$, and $P(200)$.
c) Find the domain of the function.

45. *Stopping Distance on Glare Ice.* The stopping distance (at some fixed speed) of regular tires on glare ice is a function of the air temperature F, in degrees Fahrenheit. This function is estimated by

$$D(F) = 2F + 115,$$

where $D(F)$ is the stopping distance, in feet, when the air temperature is F, in degrees Fahrenheit.

a) Graph D.
b) Find $D(0°)$, $D(-20°)$, $D(10°)$, and $D(32°)$.
c) Explain why the domain should be restricted to $[-57.5°, 32°]$.

46. *Anthropology Estimates.* Consider Example 5 and the function

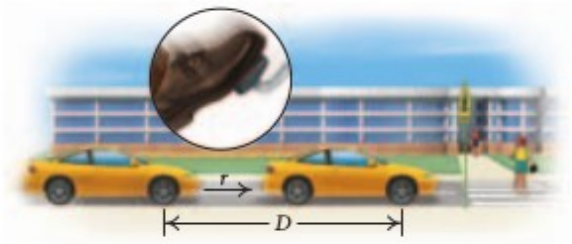
$$M(x) = 2.89x + 70.64$$

for estimating the height of a male.

a) If a 26-cm humerus from a male is found in an archeological dig, estimate the height of the male.
b) What is the domain of M?

47. *Reaction Time.* Suppose that while driving a car, you suddenly see a school crossing guard standing in the road. Your brain registers the information and sends a signal to your foot to hit the brake. The car travels a distance D, in feet, during this time, where D is a function of the speed r, in miles per hour, of the car when you see the crossing guard. That reaction distance is a linear function given by

$$D(r) = \frac{11}{10}r + \frac{1}{2}.$$



a) Find the slope of this line and interpret its meaning in this application.
b) Graph D.
c) Find $D(5)$, $D(10)$, $D(20)$, $D(50)$, and $D(65)$.
d) What is the domain of this function? Explain.

48. *Straight-Line Depreciation.* A marketing firm buys a new color printer for \$5200 to print banners for a sales campaign. The printer is purchased on January 1 and is expected to last 8 yr, at the end of which time its *trade-in*, or *salvage, value* will be \$1100. If the company figures the decline or depreciation in value to be the same each year, then the salvage value V, after t years, is given by the linear function

$$V(t) = \$5200 - \$512.50t, \quad \text{for } 0 \le t \le 8.$$

a) Graph V.
b) Find $V(0)$, $V(1)$, $V(2)$, $V(3)$, and $V(8)$.
c) Find the domain and the range of this function.

49. *Total Cost.* The Cellular Connection charges \$60 for a phone and \$29 per month under its economy plan. Write an equation that can be used to determine the total cost, $C(t)$, of operating a Cellular Connection phone for t months. Then find the total cost for 6 months.

50. *Total Cost.* Superior Cable Television charges a \$65 installation fee and \$80 per month for "deluxe" service. Write an equation that can be used to determine the total cost, $C(t)$, for t months of deluxe cable television service. Then find the total cost for 8 months of service.

In Exercises 51 and 52, the term ***fixed costs*** *refers to the start-up costs of operating a business. This includes machinery and building costs. The term* ***variable costs*** *refers to what it costs a business to produce or service one item.*

51. Kara's Custom Tees experienced fixed costs of \$800 and variable costs of \$3 per shirt. Write an equation that can be used to determine the total costs encountered by Kara's Custom Tees. Then determine the total cost of producing 75 shirts.

52. It's My Racquet experienced fixed costs of \$950 and variable costs of \$18 for each tennis racquet that is restrung. Write an equation that can be used to determine the total costs encountered by It's My Racquet. Then determine the total cost of restringing 150 tennis racquets.

Collaborative Discussion and Writing

53. Discuss why the graph of a vertical line $x = a$ cannot represent a function.

54. Explain as you would to a fellow student how the numerical value of slope can be used to describe the slant and the steepness of a line.

Skill Maintenance

If $f(x) = x^2 - 3x$, find each of the following.

55. $f\left(\frac{1}{2}\right)$

56. $f(5)$

57. $f(-5)$

58. $f(-a)$

59. $f(a + h)$

Synthesis

60. *Grade of Treadmills.* A treadmill is 5 ft long and is set at an 8% grade. How high is the end of the treadmill?

Find the slope of the line containing the given points.

61. $(-c, -d)$ and $(9c, -2d)$

62. $(r, s + t)$ and (r, s)

63. $(z + q, z)$ and $(z - q, z)$

64. $(-a - b, p + q)$ and $(a + b, p - q)$

65. (a, a^2) and $(a + h, (a + h)^2)$

66. $(a, 3a + 1)$ and $(a + h, 3(a + h) + 1)$

Suppose that f is a linear function. Determine whether each of the following is true or false.

67. $f(cd) = f(c)f(d)$

68. $f(c + d) = f(c) + f(d)$

69. $f(c - d) = f(c) - f(d)$

70. $f(kx) = kf(x)$

Let $f(x) = mx + b$. Find a formula for $f(x)$ given each of the following.

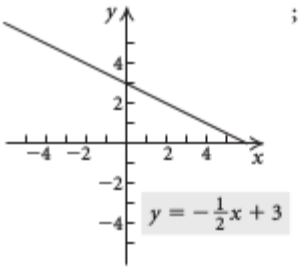
71. $f(x + 2) = f(x) + 2$

72. $f(3x) = 3f(x)$

Exercise Set 1.3

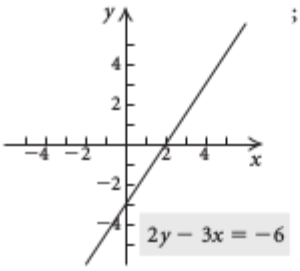
1. (a) Yes; (b) yes; (c) yes **3.** (a) Yes; (b) no; (c) no
5. $-\frac{3}{5}$ **7.** 0 **9.** $\frac{1}{5}$ **11.** $-\frac{5}{3}$ **13.** 0.3 **15.** 0
17. $-\frac{6}{5}$ **19.** $-\frac{1}{3}$ **21.** Not defined

23. ; $m = -\frac{1}{2}$



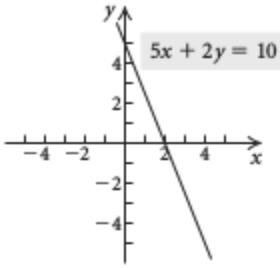
$y = -\frac{1}{2}x + 3$

25. ; $m = \frac{3}{2}$



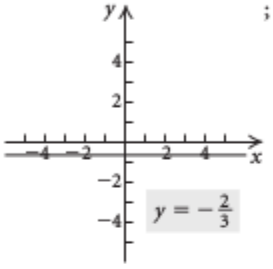
$2y - 3x = -6$

27. ; $m = -\frac{5}{2}$



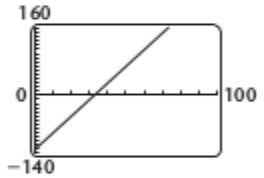
$5x + 2y = 10$

29. ; $m = 0$ **31.** 1.3

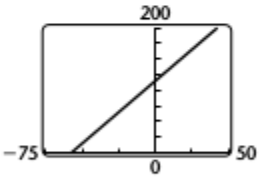


$y = -\frac{2}{3}$

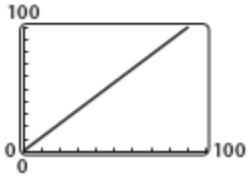
33. Not defined **35.** $-\frac{1}{10}$ **37.** The average rate of change over the 10-year period was \$566.50 per year.
39. The average rate of change over the 10-year period was about 1363 adoptions per year. **41.** $\frac{1}{6}$ km per minute
43. (a) $W(h) = 4h - 130$; (b) $y = 4x - 130$



(c) 118 lb; (d) $\{h \mid h > 32.5\}$, or $(32.5, \infty)$
45. (a) $y = 2x + 115$



(b) 115 ft, 75 ft, 135 ft, 179 ft; (c) Below −57.5°, stopping distance is negative; above 32°, ice doesn't form.
47. (a) $\frac{11}{10}$. For each mile per hour faster that the car travels, it takes $\frac{11}{10}$ ft longer to stop; (b) $y = \frac{11}{10}x + \frac{1}{2}$



(c) 6, 11.5, 22.5, 55.5, 72; (d) $\{r \mid r > 0\}$, or $(0, \infty)$. If r is allowed to be 0, the function says that a stopped car has a reaction distance of $\frac{1}{2}$ ft.

49. $C(t) = 60 + 29t$; $C(6) = \$234$
51. $C(x) = 800 + 3x$; $C(75) = \$1025$
53. Discussion and Writing **55.** [1.2] $-\frac{5}{4}$
56. [1.2] 10 **57.** [1.2] 40 **58.** [1.2] $a^2 + 3a$
59. [1.2] $a^2 + 2ah + h^2 - 3a - 3h$ **61.** $-\dfrac{d}{10c}$ **63.** 0
65. $2a + h$ **67.** False **69.** False **71.** $f(x) = x + b$

Visualizing the Graph

1. E **2.** D **3.** A **4.** J **5.** C **6.** F **7.** H
8. G **9.** B **10.** I