07-Nov-2014
Algebra and Trigonometry in-class worksheet, Chapter 1.2

Basic Definition from the textbook:

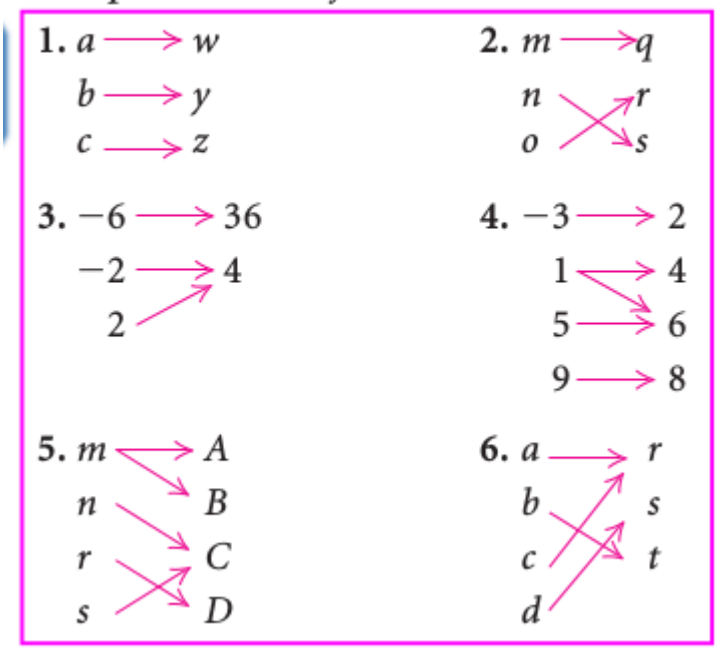
Function

A **function** is a correspondence between a first set, called the **domain**, and a second set, called the **range**, such that each member of the domain corresponds to *exactly one* member of the range.

Relation

A **relation** is a correspondence between a first set, called the **domain**, and a second set, called the **range**, such that each member of the domain corresponds to *at least one* member of the range.

In Exercises 1–14, determine whether the correspondence is a function.



Determine whether the relation is a function. Identify the domain and the range.

15. $\{(2,10),(3,15),(4,20)\}$

16. $\{(3,1),(5,1),(7,1)\}$

17. $\{(-7,3),(-2,1),(-2,4),(0,7)\}$

18. $\{(1,3),(1,5),(1,7),(1,9)\}$

19. $\{(-2,1),(0,1),(2,1),(4,1),(-3,1)\}$

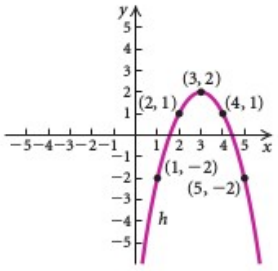
20. $\{(5,0),(3,-1),(0,0),(5,-1),(3,-2)\}$

Notation for Functions

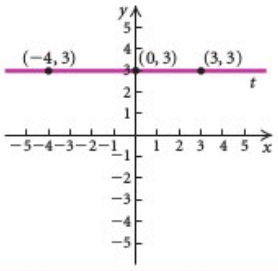
Functions used in mathematics are often given by equations. They generally require that certain calculations be performed in order to determine which member of the range is paired with each member of the domain.

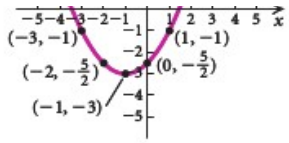
A graph of a function is shown. Find the indicated function values.

21. $h(1)$, $h(3)$, and $h(4)$

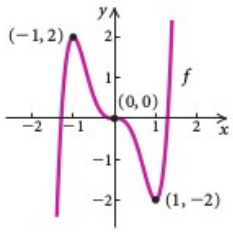


22. $t(-4)$, $t(0)$, and $t(3)$

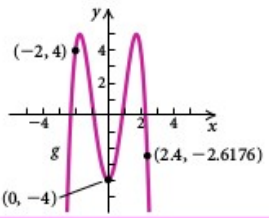




25. $f(-1)$, $f(0)$, and $f(1)$



26. $g(-2)$, $g(0)$, and $g(2.4)$



27. Given that $g(x) = 3x^2 - 2x + 1$, find each of the following.

a) $g(0)$ **b)** $g(-1)$
c) $g(3)$ **d)** $g(-x)$
e) $g(1 - t)$

28. Given that $f(x) = 5x^2 + 4x$, find each of the following.

a) $f(0)$ **b)** $f(-1)$
c) $f(3)$ **d)** $f(t)$
e) $f(t - 1)$

29. Given that $g(x) = x^3$, find each of the following.

a) $g(2)$ **b)** $g(-2)$
c) $g(-x)$ **d)** $g(3y)$
e) $g(2 + h)$

30. Given that $f(x) = 2|x| + 3x$, find each of the following.

a) $f(1)$ **b)** $f(-2)$
c) $f(-x)$ **d)** $f(2y)$
e) $f(2 - h)$

31. Given that $g(x) = \dfrac{x - 4}{x + 3}$, find each of the following.

a) $g(5)$ **b)** $g(4)$
c) $g(-3)$ **d)** $g(-16.25)$
e) $g(x + h)$

32. Given that $f(x) = \dfrac{x}{2 - x}$, find each of the following.

a) $f(2)$ **b)** $f(1)$
c) $f(-16)$ **d)** $f(-x)$
e) $f\left(-\dfrac{2}{3}\right)$

33. Find $g(0)$, $g(-1)$, $g(5)$, and $g\left(\frac{1}{2}\right)$ for

$$g(x) = \frac{x}{\sqrt{1 - x^2}}.$$

34. Find $h(0)$, $h(2)$, and $h(-x)$ for

$$h(x) = x + \sqrt{x^2 - 1}.$$

Finding Domains of Functions

When a function f, whose inputs and outputs are real numbers, is given by a formula, the *domain* is understood to be the set of all inputs for which the expression is defined as a real number. When a substitution results in an expression that is not defined as a real number, we say that the function value *does not exist* and that the number being substituted *is not* in the domain of the function.

The Vertical-Line Test

If it is possible for a vertical line to cross a graph more than once, then the graph is *not* the graph of a function.

Visualizing Domain and Range

Keep the following in mind regarding the *graph* of a function:

Domain = the set of a function's inputs, found on the horizontal axis;

Range = the set of a function's outputs, found on the vertical axis.

Find the domain of the function. Do not use a graphing calculator.

37. $f(x) = 7x + 4$

38. $f(x) = |3x - 2|$

39. $f(x) = 4 - \dfrac{2}{x}$

40. $f(x) = \dfrac{1}{x^4}$

41. $f(x) = \dfrac{x + 5}{2 - x}$

42. $f(x) = \dfrac{8}{x + 4}$

43. $f(x) = \dfrac{1}{x^2 - 4x - 5}$

44. $f(x) = \dfrac{x^4 - 2x^3 + 7}{3x^2 - 10x - 8}$

45. $f(x) = \sqrt{8 - x}$

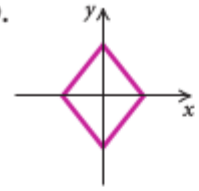
46. $f(x) = x^2 - 2x$

47. $f(x) = \frac{1}{10}|x|$

48. $f(x) = \dfrac{\sqrt{x + 1}}{x}$

In Exercises 59–66, determine whether the graph is that of a function. An open dot indicates that the point does not belong to the graph.

59.



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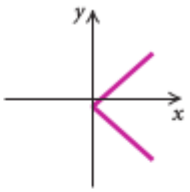

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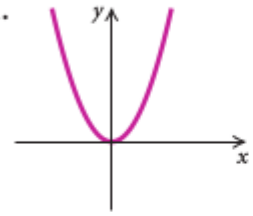

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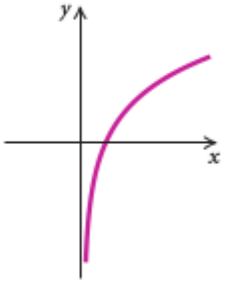

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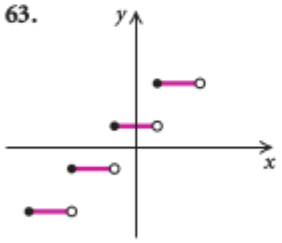

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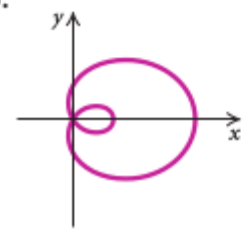

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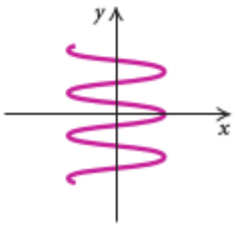
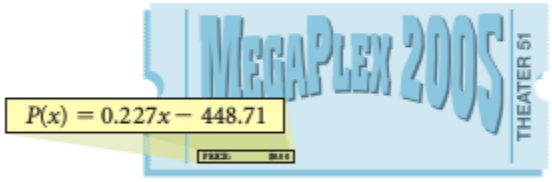


71. *Boiling Point and Elevation.* The elevation E, in meters, above sea level at which the boiling point of water is t degrees Celsius is given by the function

$$E(t) = 1000(100 - t) + 580(100 - t)^2.$$

At what elevation is the boiling point 99.5°? 100°?

72. *Average Price of a Movie Ticket.* The average price of a movie ticket, in dollars, can be estimated by the function P given by



where x is the year. Thus, $P(2005)$ is the average price of a movie ticket in 2005. The price is lower than what might be expected due to lower prices for matinees, senior citizens' discounts, and other special prices.

a) Use the function to predict the average price in 2008 and 2010.

b) When will the average price be $8.50?

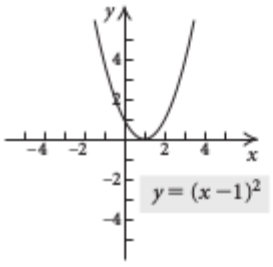
73. *Territorial Area of an Animal.* The territorial area of an animal is defined to be its defended, or exclusive, region. For example, a lion has a certain region over which it is considered ruler. It has been shown that the territorial area T, in acres, of predatory animals is a function of body weight w, in pounds, and is given by the function

$$T(w) = w^{1.31}.$$

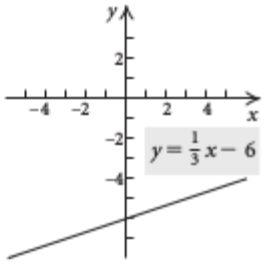
Find the territorial area of animals whose body weights are 0.5 lb, 10 lb, 20 lb, 100 lb, and 200 lb.

Exercise Set 1.2

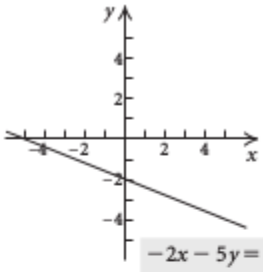
1. Yes **3.** Yes **5.** No **7.** Yes **9.** Yes **11.** Yes
13. No **15.** Function; domain: $\{2, 3, 4\}$; range: $\{10, 15, 20\}$
17. Not a function; domain: $\{-7, -2, 0\}$; range: $\{3, 1, 4, 7\}$
19. Function; domain: $\{-2, 0, 2, 4, -3\}$; range: $\{1\}$
21. $h(1) = -2$; $h(3) = 2$; $h(4) = 1$
23. $s(-4) = 3$; $s(-2) = 0$; $s(0) = -3$
25. $f(-1) = 2$; $f(0) = 0$; $f(1) = -2$
27. **(a)** 1; **(b)** 6; **(c)** 22; **(d)** $3x^2 + 2x + 1$; **(e)** $3t^2 - 4t + 2$
29. **(a)** 8; **(b)** -8; **(c)** $-x^3$; **(d)** $27y^3$; **(e)** $8 + 12h + 6h^2 + h^3$ **31.** **(a)** $\frac{1}{8}$; **(b)** 0; **(c)** does not exist; **(d)** $\frac{81}{53}$, or approximately 1.5283; **(e)** $\frac{x + h - 4}{x + h + 3}$
33. 0; does not exist; does not exist as a real number; $\frac{1}{\sqrt{3}}$, or $\frac{\sqrt{3}}{3}$ **35.** $g(-2.1) \approx -21.8$; $g(5.08) \approx -130.4$; $g(10.003) \approx -468.3$ **37.** All real numbers, or $(-\infty, \infty)$
39. $\{x \mid x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$
41. $\{x \mid x \neq 2\}$, or $(-\infty, 2) \cup (2, \infty)$
43. $\{x \mid x \neq -1 \text{ and } x \neq 5\}$, or $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$
45. $\{x \mid x \leq 8\}$, or $(-\infty, 8]$ **47.** All real numbers, or $(-\infty, \infty)$ **49.** Domain: all real numbers; range: $[0, \infty)$
51. Domain: $[-3, 3]$; range: $[0, 3]$
53. Domain: all real numbers; range: all real numbers
55. Domain: $(-\infty, 7]$; range: $[0, \infty)$
57. Domain: all real numbers; range: $(-\infty, 3]$
59. No **61.** Yes **63.** Yes **65.** No
67. Domain: $[0, 5]$; range: $[0, 3]$
69. Domain: $[-2\pi, 2\pi]$; range: $[-1, 1]$
71. 645 m; 0 m **73.** 0.4 acre; 20.4 acres; 50.6 acres; 416.9 acres; 1033.6 acres **75.** Discussion and Writing
76. [1.1] $(-3, -2)$, yes; $(2, -3)$, no
77. [1.1] $(0, -7)$, no; $(8, 11)$, yes
78. [1.1] $\left(\frac{4}{5}, -2\right)$, yes; $\left(\frac{11}{5}, \frac{1}{10}\right)$, yes
79. [1.1]



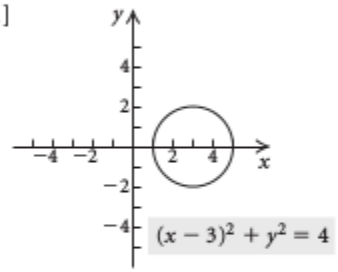
80. [1.1]



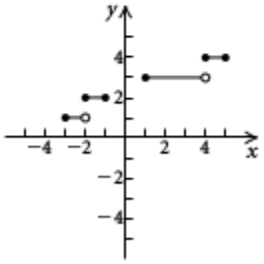
81. [1.1]



82. [1.1]



83. $f(x) = x$, $g(x) = x + 1$ **85.**



87. -7 **89.** **(a)** $f(x) = -2x + 1$; **(b)** $f(x) = 1$; **(c)** $f(x) = 2x - 1$