Algebra, Chapter 1.1

Class Road-Map:

~~Chapter R~~

College Algebra	College Trigonometry
Ch 1) Graphs, Functions, and Models	Ch 5) The Trigonometric Functions
Ch 2) Functions, Equations, and Inequalities	Ch 6) Identities, Inverse Functions, Equations
Ch 3) Polynomial and Rational Functions	Ch 7) Applications of Trigonometry
Ch 4) Exponential and Logarithmic Functions	

Supplemental
Ch 8) Systems of Equations and Matrices
Ch 9) Analytic Geometry Topics
Ch 10) Sequences, Series, and Combinatorics

Chapter 1 topics:

1.1 Introduction to Graphing
1.2 Functions and Graphs
1.3 Linear Functions, Slope, and Applications
1.4 Equations of Lines and Modeling
1.5 More on Functions
1.6 The Algebra of Functions
1.7 Symmetry and Transformations

Chapter 1 Review Sheet: the B.L.U.F. ("*Bottom Line Up Front*")

Chapter 1 Summary and Review

Important Properties and Formulas

The Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Terminology about Lines

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

The Slope–Intercept Equation: $y = mx + b$

The Point–Slope Equation: $y - y_1 = m(x - x_1)$

Horizontal Line: $y = b$

Vertical Line: $x = a$

Parallel Lines: $m_1 = m_2, \; b_1 \neq b_2$

Perpendicular Lines: $m_1 m_2 = -1$, or $x = a, y = b$

The Algebra of Functions

The Sum of Two Functions:
$(f + g)(x) = f(x) + g(x)$

The Difference of Two Functions:
$(f - g)(x) = f(x) - g(x)$

The Product of Two Functions:
$(fg)(x) = f(x) \cdot g(x)$

The Quotient of Two Functions:
$(f/g)(x) = f(x)/g(x), \; g(x) \neq 0$

The Composition of Two Functions:
$(f \circ g)(x) = f(g(x))$

Tests for Symmetry

x-axis: If replacing y with $-y$ produces an equivalent equation, then the graph is symmetric with respect to the x-axis.

y-axis: If replacing x with $-x$ produces an equivalent equation, then the graph is symmetric with respect to the y-axis.

Origin: If replacing x with $-x$ and y with $-y$ produces an equivalent equation, then the graph is symmetric with respect to the origin.

Even Function: $f(-x) = f(x)$

Odd Function: $f(-x) = -f(x)$

Transformations

Vertical Translation: $y = f(x) \pm b$

Horizontal Translation: $y = f(x \mp d)$

Reflection across the x-axis: $y = -f(x)$

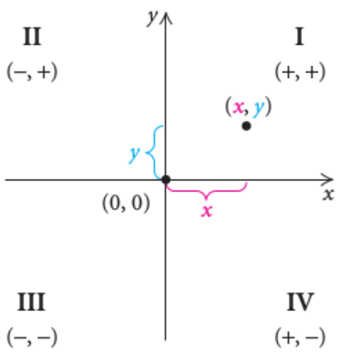
Reflection across the y-axis: $y = f(-x)$

Vertical Stretching or Shrinking: $y = af(x)$

Horizontal Stretching or Shrinking: $y = f(cx)$

Chapter 1.1 Introduction to Graphing

- Plot points.
- Determine whether an ordered pair is a solution of an equation.
- Graph equations.
- Find the distance between two points in the plane and find the midpoint
- of a segment.
- Find an equation of a circle with a given center and radius, and given an
- equation of a circle, find the center and the radius.
- Graph equations of circles.



Many real-world situations can be modeled, or described mathematically, using equations in which two variables appear. We use a plane to graph a pair of numbers. To locate points on a plane, we use two perpendicular number lines, called **axes**, which intersect at (0,0). We call this point the **origin**. The horizontal axis is called the ***x*-axis**, and the vertical axis is called the ***y*-axis**. (Other variables, such as *a* and *b*, can also be used.) The axes divide the plane into four regions, called **quadrants**, denoted by Roman numerals and numbered counterclockwise from the upper right. Arrows show the positive direction of each axis.

Each point (x, y) in the plane is described by an **ordered pair**. The first number, x, indicates the point's horizontal location with respect to the y-axis, and the second number, y, indicates the point's vertical location with respect to the x-axis. We call x the **first coordinate**, ***x*-coordinate**, or **abscissa**. We call y the **second coordinate**, ***y*-coordinate**, or **ordinate**. Such a representation is called the **Cartesian coordinate system** in honor of the French mathematician and philosopher René Descartes (1596–1650).

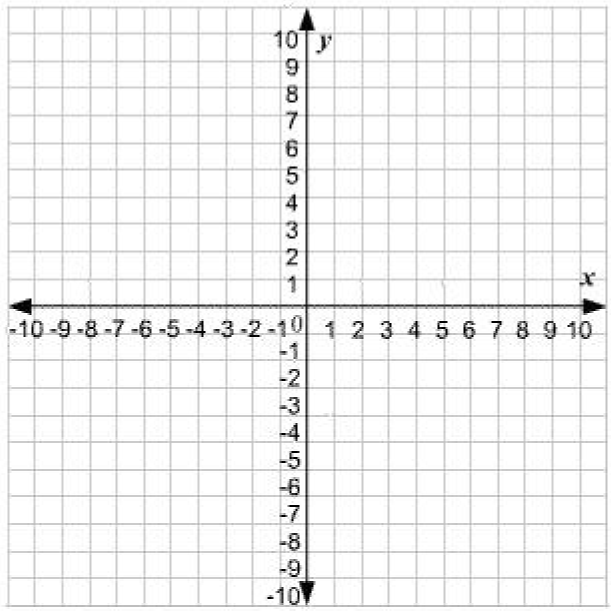
Graph and label the given points by hand.

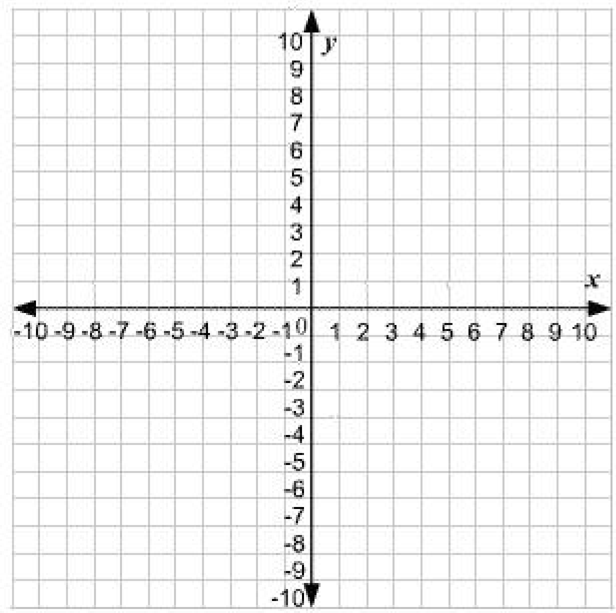
1. $(4, 0), (-3, -5), (-1, 4), (0, 2), (2, -2)$

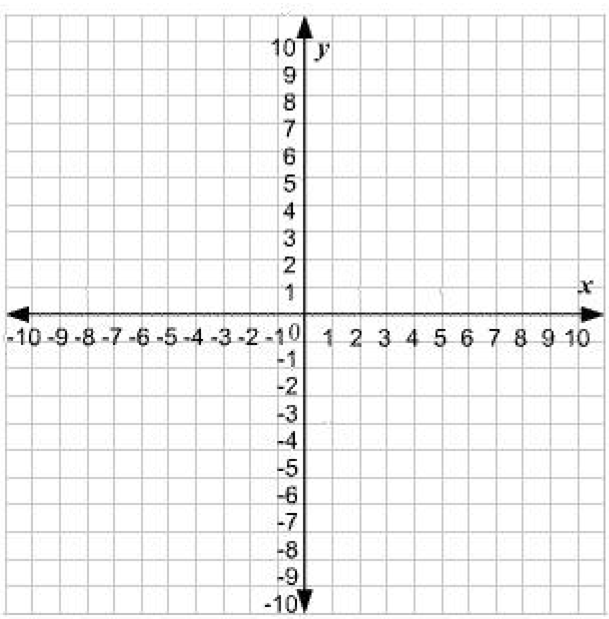
2. $(1, 4), (-4, -2), (-5, 0), (2, -4), (4, 0)$

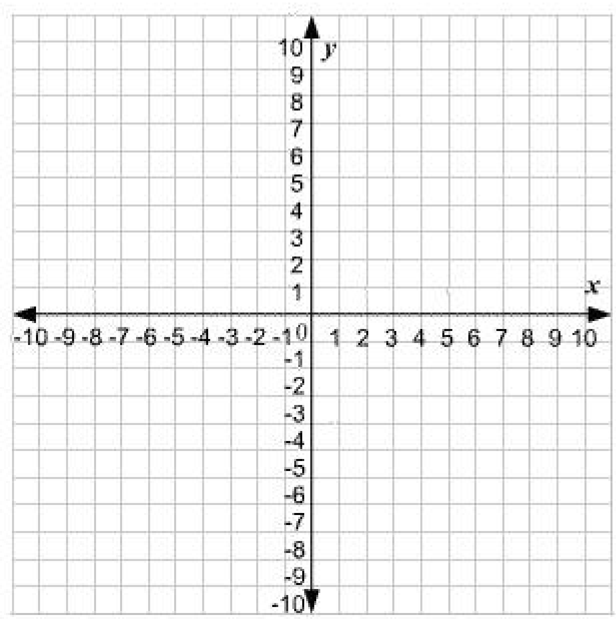
3. $(-5, 1), (5, 1), (2, 3), (2, -1), (0, 1)$

4. $(4, 0), (4, -3), (-5, 2), (-5, 0), (-1, -5)$









Use substitution to determine whether the given ordered pairs are solutions of the given equation.

7. $(1,-1)$, $(0,3)$; $y = 2x - 3$

8. $(2,5)$, $(-2,-5)$; $y = 3x - 1$

9. $\left(\frac{2}{3},\frac{3}{4}\right)$, $\left(1,\frac{3}{2}\right)$; $6x - 4y = 1$

10. $(1.5, 2.6)$, $(-3,0)$; $x^2 + y^2 = 9$

11. $\left(-\frac{1}{2},-\frac{4}{5}\right)$, $\left(0,\frac{3}{5}\right)$; $2a + 5b = 3$

12. $\left(0,\frac{3}{2}\right)$, $\left(\frac{2}{3},1\right)$; $3m + 4n = 6$

13. $(-0.75, 2.75)$, $(2,-1)$; $x^2 - y^2 = 3$

14. $(2,-4)$, $(4,-5)$; $5x + 2y^2 = 70$

Use a graphing calculator to create a table of values with TBLSTART $= -3$ *and* ΔTBL $= 1$. *Then graph the equation by hand.*

15. $y = 3x + 5$

16. $y = -2x - 1$

17. $x - y = 3$

18. $x + y = 4$

19. $2x + y = 4$

20. $3x - y = 6$

21. $y = -\frac{3}{4}x + 3$

22. $3y - 2x = 3$

23. $5x - 2y = 8$

24. $y = 2 - \frac{4}{3}x$

25. $x - 4y = 5$

26. $6x - y = 4$

27. $3x - 4y = 12$

28. $2x + 3y = -6$

29. $2x + 5y = -10$

30. $4x - 3y = 12$

31. $y = -x^2$

32. $y = x^2$

33. $y = x^2 - 3$

34. $y = 4 - x^2$

35. $y = -x^2 + 2x + 3$

36. $y = x^2 + 2x - 1$

x- and y-Intercepts

An ***x*-intercept** is a point $(a, 0)$. To find a, let $y = 0$ and solve for x.

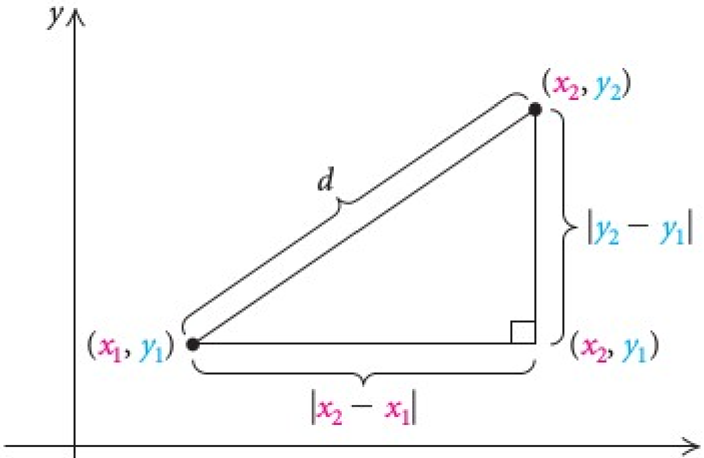
A ***y*-intercept** is a point $(0, b)$. To find b, let $x = 0$ and solve for y.

x	y

x	y

x	y

x	y



The Distance Formula

The **distance** d between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Find the distance between the pair of points. Give an exact answer and, where appropriate, to three decimal places.

59. $(4, 6)$ and $(5, 9)$

60. $(-3$

61. $(6, -1)$ and $(9, 5)$

62. $(-4$

63. $(-4.2, 3)$ and $(2.1, -6.4)$

64. $\left(-\frac{3}{5}, -4\right)$ and $\left(-\frac{3}{5}, \frac{2}{3}\right)$

65. $\left(-\frac{1}{2}, 4\right)$ and $\left(\frac{5}{2}, 4\right)$

66. $(0.6, -1.5)$ and $(-8.1, -1.5)$

67. $\left(\sqrt{3}, -\sqrt{5}\right)$ and $\left(-\sqrt{6}, 0\right)$

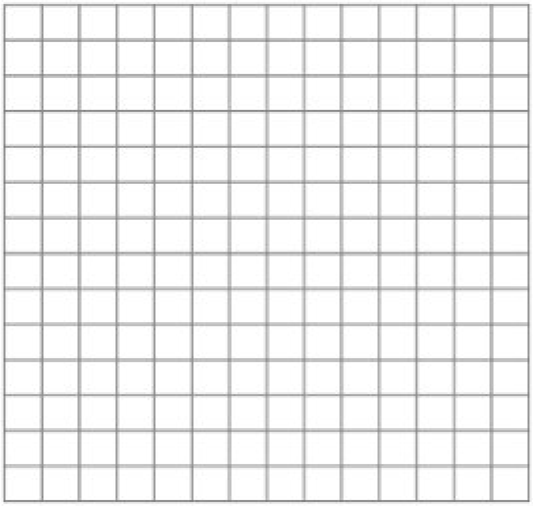
68. $\left(-\sqrt{2}, 1\right)$ and $\left(0, \sqrt{7}\right)$

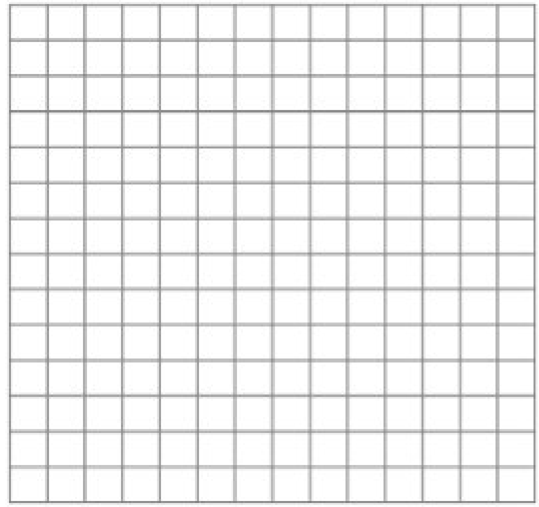
69. $(0, 0)$ and (a, b)

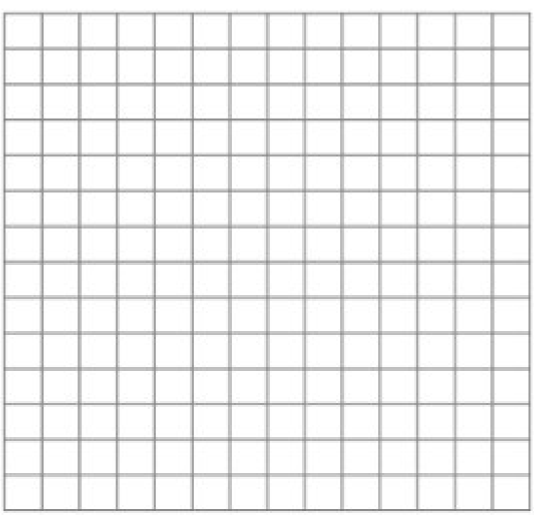
70. (r, s) and $(-r, -s)$

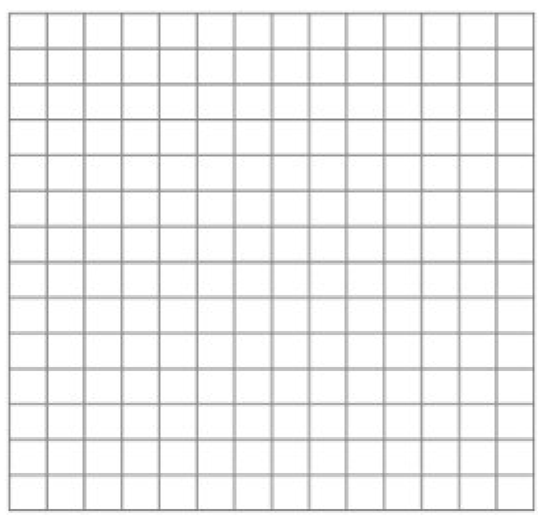
71. The points $(-3, -1)$ and $(9, 4)$ are the endpoints of the diameter of a circle. Find the length of the radius of the circle.

72. The point $(0, 1)$ is on a circle that has center $(-3, 5)$. Find the length of the diameter of the circle.





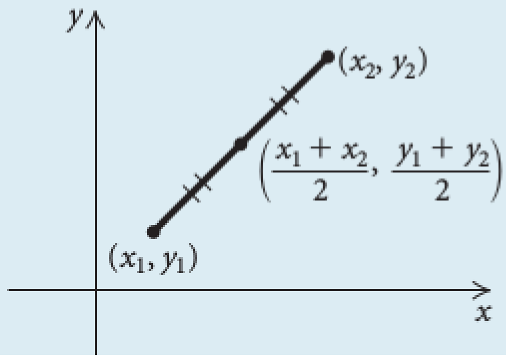




The Midpoint Formula

If the endpoints of a segment are (x_1, y_1) and (x_2, y_2), then the coordinates of the **midpoint** are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



Find the midpoint of the segment having the given endpoints.

77. $(4, -9)$ and $(-12, -3)$

78. $(7, -2)$ and $(9, 5)$

79. $(6.1, -3.8)$ and $(3.8, -6.1)$

80. $(-0.5, -2.7)$ and $(4.8, -0.3)$

81. $(-6, 5)$ and $(-6, 8)$

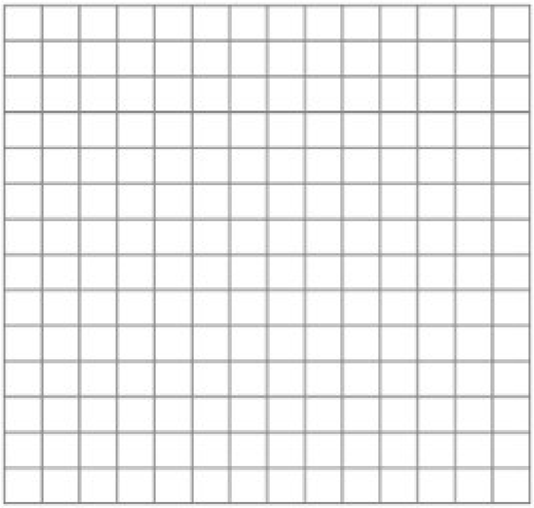
82. $(1, -2)$ and $(-1, 2)$

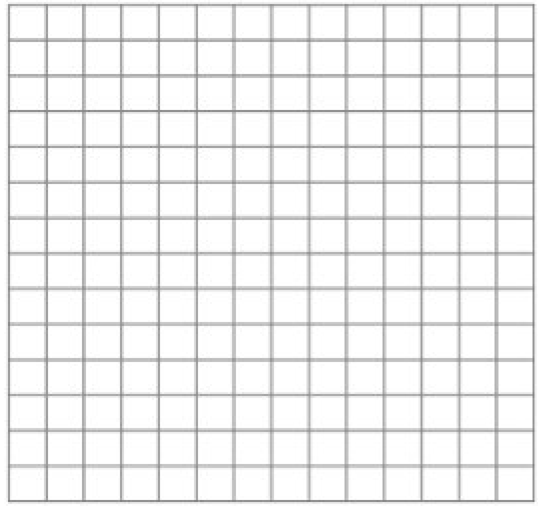
83. $\left(-\frac{1}{6}, -\frac{3}{5}\right)$ and $\left(-\frac{2}{3}, \frac{5}{4}\right)$

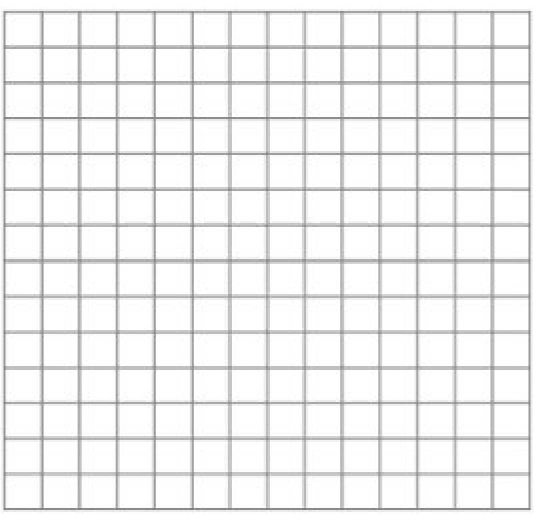
84. $\left(\frac{2}{9}, \frac{1}{3}\right)$ and $\left(-\frac{2}{5}, \frac{4}{5}\right)$

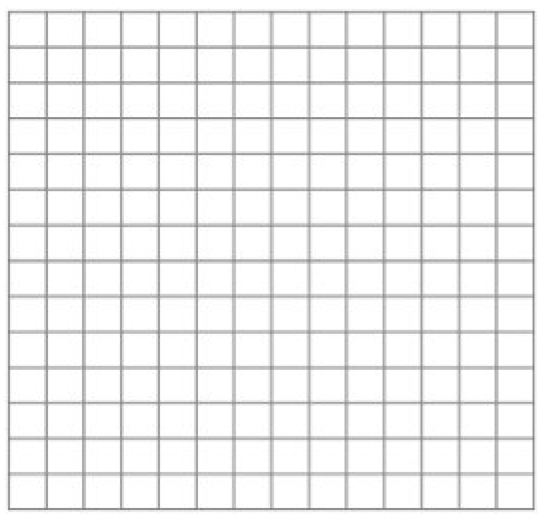
85. $\left(\sqrt{3}, -1\right)$ and $\left(3\sqrt{3}, 4\right)$

86. $\left(-\sqrt{5}, 2\right)$ and $\left(\sqrt{5}, \sqrt{7}\right)$









The Equation of a Circle

The equation of a circle with center (h, k) and radius r, in standard form, is

$$(x - h)^2 + (y - k)^2 = r^2.$$

EXAMPLE 12 Find the center and the radius of the circle and graph it:

$$x^2 + y^2 - 8x + 2y + 13 = 0.$$

Solution First, we regroup the terms in order to **complete the square** twice, once with $x^2 - 8x$ and once with $y^2 + 2y$:

$$x^2 + y^2 - 8x + 2y + 13 = 0$$

$$(x^2 - 8x) + (y^2 + 2y) + 13 = 0. \qquad \text{Regrouping}$$

Next, we complete the square inside each set of parentheses. We want to add something to both $x^2 - 8x$ and $y^2 + 2y$ so that each becomes the square of a binomial. For $x^2 - 8x$, we take half the x-coefficient, $\frac{1}{2}(-8) = -4$, and square it, $(-4)^2 = 16$. Then we add 0, or $16 - 16$, inside the parentheses. For $y^2 + 2y$, we have $\frac{1}{2} \cdot 2 = 1$ and $1^2 = 1$, so we add $1 - 1$ inside the parentheses.

$$(x^2 - 8x + 0) + (y^2 + 2y + 0) + 13 = 0 \qquad \text{Adding 0}$$

$$(x^2 - 8x + 16 - 16) + (y^2 + 2y + 1 - 1) + 13 = 0$$

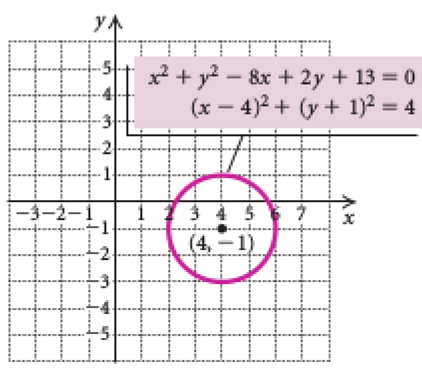
$$(x^2 - 8x + 16) + (y^2 + 2y + 1) - 16 - 1 + 13 = 0 \qquad \text{Regrouping}$$

$$(x - 4)^2 + (y + 1)^2 - 4 = 0 \qquad \text{Factoring and simplifying}$$

$$(x - 4)^2 + (y + 1)^2 = 4 \qquad \text{Adding 4}$$

$$(x - 4)^2 + [y - (-1)]^2 = 2^2 \qquad \text{Writing in standard form}$$

The center is $(4, -1)$ and the radius is 2.



Find the center and the radius of the circle. Then graph the circle by hand. Check your graph with a graphing calculator.

101. $x^2 + y^2 = 4$

102. $x^2 + y^2 = 81$

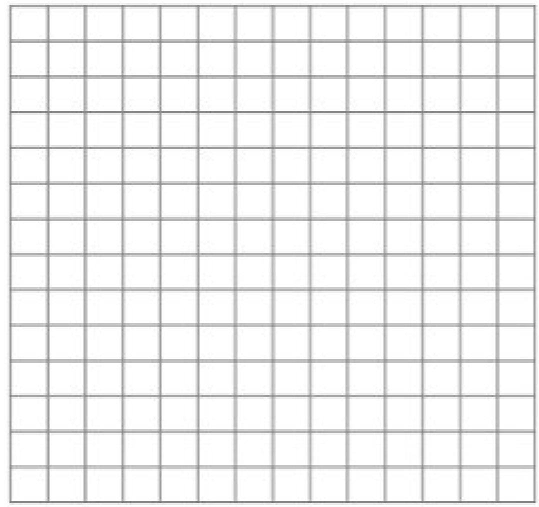
103. $x^2 + (y - 3)^2 = 16$

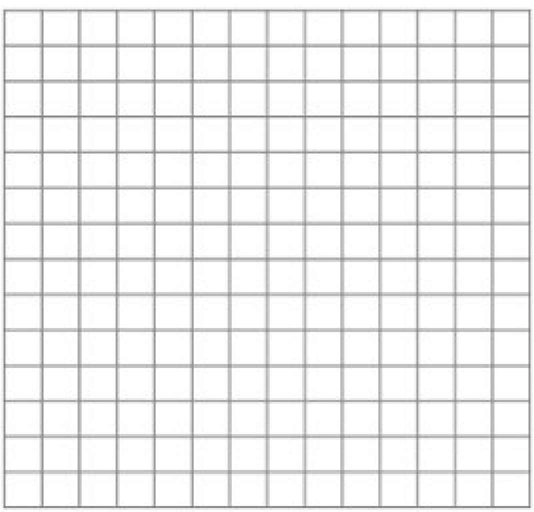
104. $(x + 2)^2 + y^2 = 100$

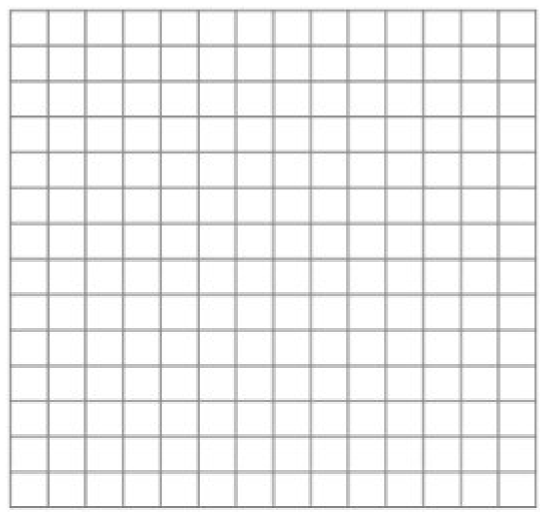
105. $(x - 1)^2 + (y - 5)^2 = 36$

106. $(x - 7)^2 + (y + 2)^2 = 25$

107. $(x + 4)^2 + (y + 5)^2 = 9$







Find an equation for a circle satisfying the given conditions.

93. Center $(2, 3)$, radius of length $\frac{5}{3}$

94. Center $(4, 5)$, diameter of length 8.2

95. Center $(-1, 4)$, passes through $(3, 7)$

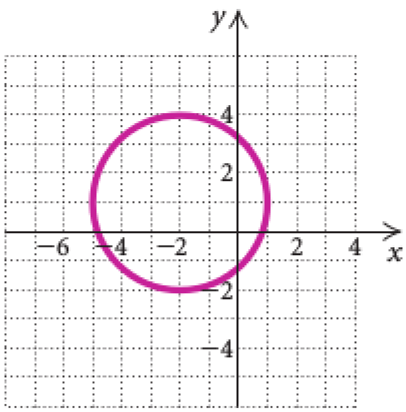
96. Center $(6, -5)$, passes through $(1, 7)$

97. The points $(7, 13)$ and $(-3, -11)$ are at the ends a diameter.

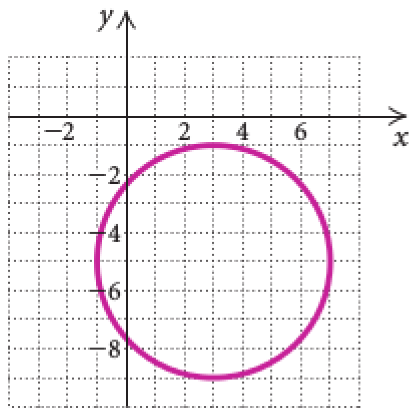
98. The points $(-9, 4)$, $(-2, 5)$, $(-8, -3)$, and $(-1, -2)$ are vertices of an inscribed square.

Find the equation of the circle. Express the equation in standard form.

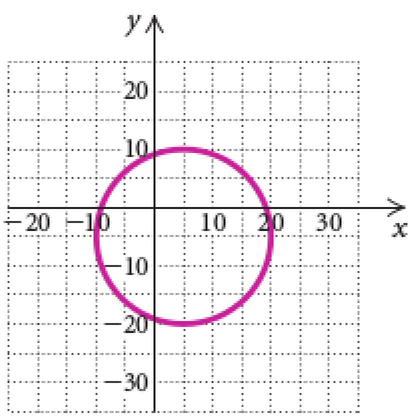
113.



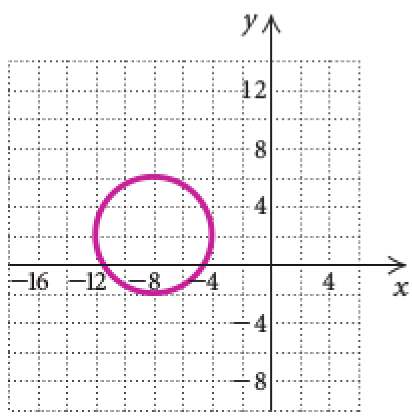
114.



115.



116.



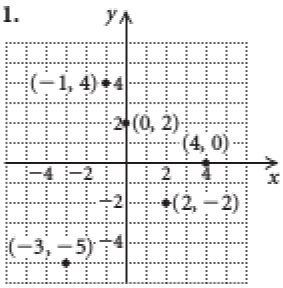
Chapter 1

Visualizing the Graph

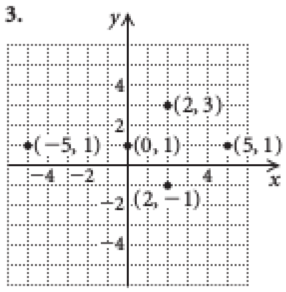
1. H **2.** B **3.** D **4.** A **5.** G **6.** I **7.** C
8. J **9.** F **10.** E

Exercise Set 1.1

1.

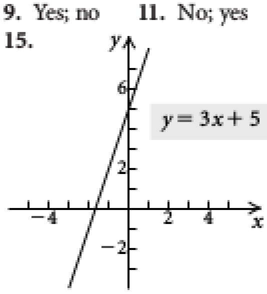


3.

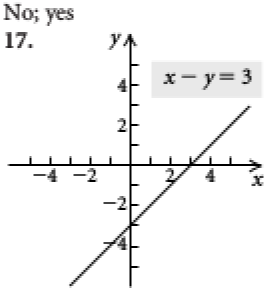


5. (1999, \$19 billion), (2000, \$25 billion), (2001, \$32 billion), (2002, \$36 billion), (2003, \$43 billion) **7.** Yes; no
9. Yes; no **11.** No; yes **13.** No; yes

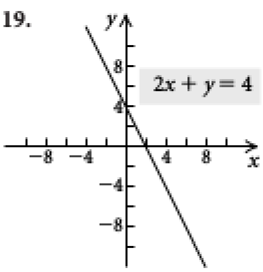
15.



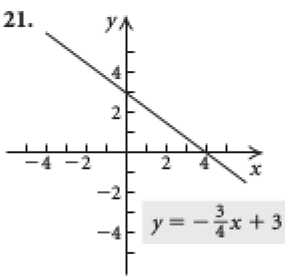
17.



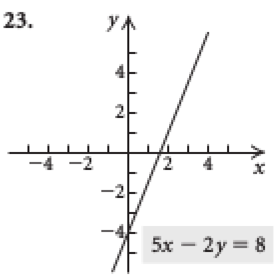
19.



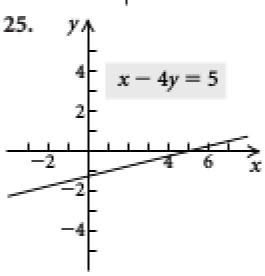
21.



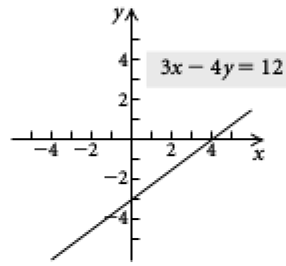
23.



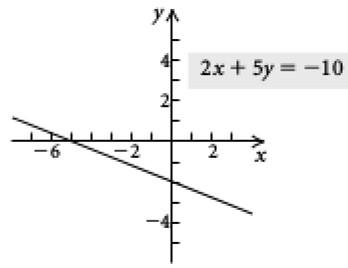
25.



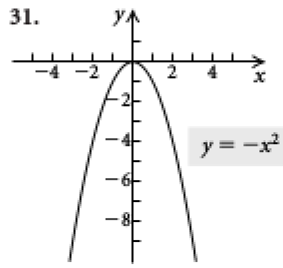
27.



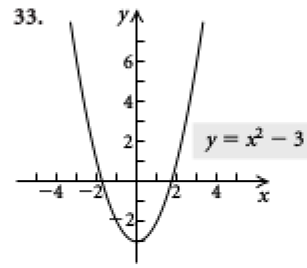
29.



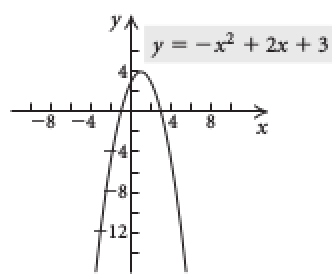
31.



33.

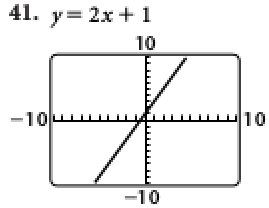


35.

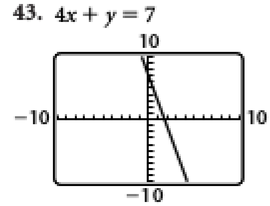


37. (b) **39.** (a)

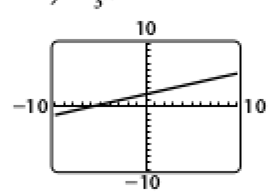
41. $y = 2x + 1$



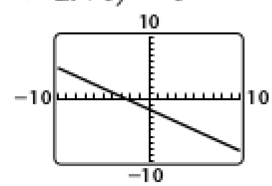
43. $4x + y = 7$



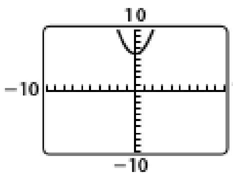
45. $y = \frac{1}{3}x + 2$



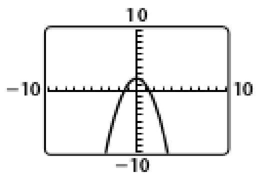
47. $2x + 3y = -5$



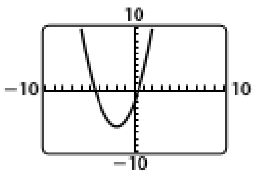
49. $y = x^2 + 6$



51. $y = 2 - x^2$

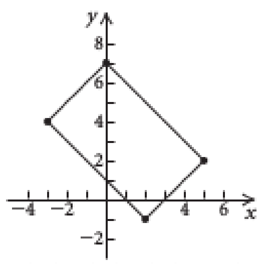


53. $y = x^2 + 4x - 2$



55. Standard window

57. $[-1, 1, -0.3, 0.3]$ **59.** $\sqrt{10}$, 3.162 **61.** $\sqrt{45}$, 6.708
63. $\sqrt{128.05}$, 11.316 **65.** 3 **67.** $\sqrt{14 + 6\sqrt{2}}$, 4.742
69. $\sqrt{a^2 + b^2}$ **71.** 6.5 **73.** Yes **75.** No
77. $(-4, -6)$ **79.** $(4.95, -4.95)$ **81.** $\left(-6, \frac{13}{2}\right)$
83. $\left(-\frac{5}{12}, \frac{13}{40}\right)$ **85.** $\left(2\sqrt{3}, \frac{3}{2}\right)$
87.

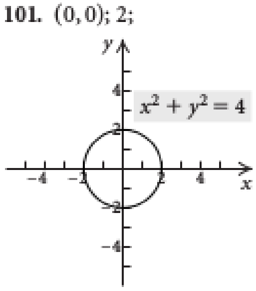


$\left(-\frac{1}{2}, \frac{3}{2}\right), \left(\frac{7}{2}, \frac{1}{2}\right), \left(\frac{5}{2}, \frac{9}{2}\right), \left(-\frac{3}{2}, \frac{11}{2}\right)$; no

89. $\left(\dfrac{\sqrt{7} + \sqrt{2}}{2}, -\dfrac{1}{2}\right)$ **91.** Square the window; for example, $[-12, 9, -4, 10]$.

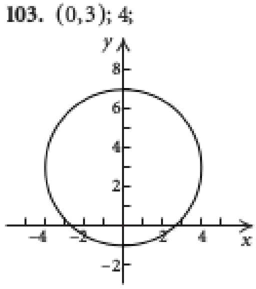
93. $(x - 2)^2 + (y - 3)^2 = \dfrac{25}{9}$
95. $(x + 1)^2 + (y - 4)^2 = 25$
97. $(x - 2)^2 + (y - 1)^2 = 169$
99. $(x + 2)^2 + (y - 3)^2 = 4$

101. $(0, 0)$; 2;



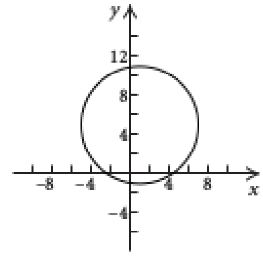
$x^2 + y^2 = 4$

103. $(0, 3)$; 4;



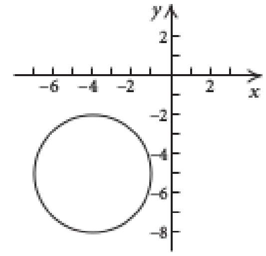
$x^2 + (y - 3)^2 = 16$

105. $(1, 5)$; 6;



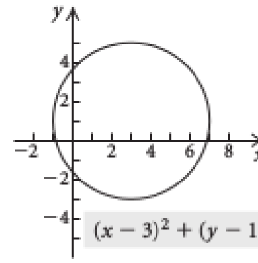
$(x - 1)^2 + (y - 5)^2 = 36$

107. $(-4, -5)$; 3;



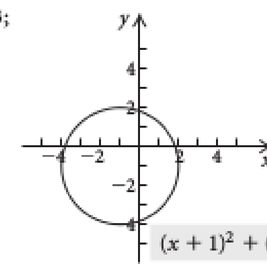
$(x + 4)^2 + (y + 5)^2 = 9$

109. $(3, 1)$; 4;



$(x - 3)^2 + (y - 1)^2 = 16$

111. $(-1, -1)$; 3;



$(x + 1)^2 + (y + 1)^2 = 9$

113. $(x + 2)^2 + (y - 1)^2 = 3^2$
115. $(x - 5)^2 + (y + 5)^2 = 15^2$
117. Discussion and Writing **119.** Third
121. $\sqrt{h^2 + h + 2a - 2\sqrt{a^2 + ah}}$, $\left(\dfrac{2a + h}{2}, \dfrac{\sqrt{a} + \sqrt{a + h}}{2}\right)$
123. $(x - 2)^2 + (y + 7)^2 = 36$ **125.** $(0, 4)$
127. $a_1 \approx 2.7$ ft, $a_2 \approx 37.3$ ft **129.** Yes **131.** Yes
133. Let $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$, and $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$. Let $d(AB)$ denote the distance from point A to point B.

(a) $$d(P_1M) = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2} = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2};$$

$$d(P_2M) = \sqrt{\left(\frac{x_1 + x_2}{2} - x_2\right)^2 + \left(\frac{y_1 + y_2}{2} - y_2\right)^2} = \frac{1}{2}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d(P_1M).$$